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Entanglement dynamics of the two-spin system with time-dependent Heisenberg and Dzyaloshinskii-Moriya interactions

Scientific research paper

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ABSTRACT

In the present paper, we study the entanglement dynamics of a two-spin system with Heisenberg interaction and Dzyaloshinskii-Moriya (DM) interaction. We assume that both interior interactions of the system are time-dependent. We consider two different scenarios: In the first case, both Heisenberg and DM interactions are sinusoidal function ($\sin(\omega t)$). The findings show that if time-dependency of both interior interactions is the same, the quantum entanglement of the system presents more robustness and its temporal fluctuations reduce significantly over time. In the second case, DM interaction is considered as $D \propto \cos(\omega t)$. The findings imply that this kind of time-dependency causes the strong fluctuations of the entanglement, so that sometimes the correlation reaches zero. Therefore, in order to achieve a more stable entanglement over time, it is better that the time dependency of the Heisenberg interaction and DM interaction would be the same.

1 Introduction

Quantum entanglement distinguishes the quantum world from the classical world. The entangled states represent a kind of non-localized quantum correlation between subsystems and is the foundation of the quantum information, quantum computation [1,2,3], quantum cryptography [4], teleportation [5], and quantum dense coding [6]. Therefore, understanding and analyzing the effects and results of entanglement and experimental applications of this feature is very attractive for physicists. Two-spin systems are the simplest quantum systems that can be entangled nowadays. Heisenberg's model is an ideal candidate for generating and manipulating entangled states, and also can be used in quantum computing processes. Therefore, many physical systems such as core spins [7], quantum dots [8], superconductors [9], and optical networks [10] have been simulated by this model. The spin chains whose interaction between their spins includes all the various kinds of Heisenberg models such as XX, XY, XZ, XXZ, etc. have been extensively studied [11-15]. In addition to the Heisenberg interaction, there is another important interaction that originates from the electron-momentum coupling. This antisymmetric spin exchange interaction that is known as the DM interaction, plays an important role in physics of low-dimensional quantum magnets [16,17]. Studies have indicated that the DM interaction can improve the dynamics behavior of the entanglement in the spin systems and prevents the sudden death of the

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entanglement due to contact of the system with environment [18,19,20,21,22,23]. In fact, the DM interaction is recognized to be very useful for both weak ferromagnetism and anti-ferromagnetism spin arrangements in low symmetry [24].

Internal interactions of the spin systems (such as DM and Heisenberg interactions) can have temporal variations because they are dependent on some features such as the temperature that may change over the time. Moreover, the time dependency of the spin-spin interaction can be due to the system constantly interacting with its surrounding environment. In some cases, the nature of the crystals can lead to time-dependent spin-spin interactions. In recent research, the effect of time-dependent DM interaction and Heisenberg interaction [25,26,27] has been investigated. There are various experimental studies in the field of quantum entanglement for different exchange interactions [28, 29, 30].

The interesting results of the mentioned papers led us to study the simultaneous effect of these two time-varying interactions on the entanglement dynamics of the system with XXX model and added DM interaction. We expect to obtain a better understanding of the time evolution of the entangled states in a two-qubit system. It is also expected that by adding the time-varying spin-spin interactions, we can find better ways to protect the system against the decoherence induced by environment, and thus, preserve entanglement from undesired environmental effects.

We structured the paper as follows: section 2 introduces the model of the spin system and calculates the entanglement evaluation of the system over time using the concurrence function. Section 3 presents the analytical discussion of the system with different time-dependent interactions. Also, the findings and results are analyzed and compared. Finally, the conclusion is presented in section 4.

2 Model

We consider a two-spin XXX Heisenberg model with Dzyaloshinsky-Mooriya interaction. The Hamiltonian of the system is given by

$$H=\frac{J}{2}(\sigma_1^x\sigma_2^x+\sigma_1^y\sigma_2^y+\sigma_1^z\sigma_2^z)+\mathrm{D}(\sigma_1^x\sigma_2^y-\sigma_1^y\sigma_2^x), \quad (1)$$

Where σ^j (j=x, y, z) are the Pauli operators, J is the strength of the Heisenberg interaction, in all three directions of x, y, z, D stands for the DM coupling vector between two qubits. We consider DM interaction along the z axis. This kind of interaction causes the effects of anisotropic antisymmetric spin-orbit interactions. In the system simple calculations show that the energy levels and eigen-vectors are:

$$\begin{aligned}
&\varepsilon_1=x\,, && |1\rangle=|\uparrow\uparrow\rangle\\
&\varepsilon_2=x\,, && |2\rangle=|\downarrow\downarrow\rangle\\
&\varepsilon_3=-x+y\,, && |3\rangle=\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle+e^{i\theta}|\downarrow\uparrow\rangle)\,, \quad (2)\\
&\varepsilon_4=-x-y\,, && |4\rangle=\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle-e^{i\theta}|\downarrow\uparrow\rangle)
\end{aligned}$$

where $x=\frac{J}{2}$, $y=\sqrt{x^2+D^2}$, $\theta=\tan^{-1}(\frac{D}{J})$.

Now, we calculate the dynamic behavior of the two-spin system. For this purpose, we use the time-evolution operator

$$U(t)=exp(-iHt/\hbar)\rightarrow\psi(t)=U(t)\psi(0)$$
$$\rightarrow\psi(t)=\frac{1}{\sqrt{2}}e^{-\frac{iHt}{\hbar}}(|01\rangle+|10\rangle). \quad (3)$$

Substituting the Hamiltonian Matrix, the elements of the time-evolution matrix can be achieved

$$\begin{aligned}
&U_{11}=U_{44}=e^{-ixt}\\
&U_{22}=U_{33}=e^{ixt}\cos yt\\
&U_{23}=-ie^{-i\theta}e^{ixt}\sin yt\\
&U_{32}=-ie^{i\theta}e^{ixt}\sin yt
\end{aligned} \quad . \quad (4)$$

The rest of the elements are zero. Let us consider the initial state of the system as

$$|\psi(0)\rangle=\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle). \quad (5)$$

Therefore, the density matrix at t=0 is given by

$$\rho(0)=|\psi(0)\rangle\langle\psi(0)|=\frac{1}{2}\begin{pmatrix}0&0&0&0\\0&1&1&0\\0&1&1&0\\0&0&0&0\end{pmatrix}, \quad (6)$$

where |0>=|↓> and |1>=|↑>. Applying the time-evolution operator on the initial density operator, we will have

$$\begin{aligned}
&\rho(t)=U(t)\rho(0)U^{\dagger}(t)\\
&\rho_{22}=\frac{1}{2}(1-\sin 2yt\sin\theta)\\
&\rho_{33}=\frac{1}{2}(1+\sin 2yt\sin\theta)\\
&\rho_{23}=\frac{1}{2}(\cos^2 yt+e^{-2i\theta}\sin^2 yt)\\
&\rho_{32}=\frac{1}{2}(\cos^2 yt+e^{2i\theta}\sin^2 yt)
\end{aligned} \quad . \tag{7}$$

We choose the concurrence [31] as a measurement of the pairwise entanglement. For a two-qubit pure or mixed state which is described by the density matrix ρ, the concurrence C is calculated as

$$C=\max\{0,\lambda_1-\lambda_2-\lambda_3-\lambda_4\}. \tag{8}$$

For a two-qubit state, λ_i's are the eigenvalues in decreasing order of the Hermitian Matrix of R which is defined as

$$R^2=\rho\tilde{\rho}, \tag{9}$$

where $\tilde{\rho}$ is the spin-flipped state defined as: $\tilde{\rho}=(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y)$. The eigenvalues of the R matrix are

$$\begin{aligned}
&\lambda_1'=\lambda_2'=\lambda_3'=0\\
&\lambda_4'=|\rho_{23}|+\sqrt{\rho_{22}\rho_{33}}=\sqrt{1-\sin^2 2yt\sin^2\theta}
\end{aligned} . \tag{10}$$

Therefore, the concurrence of the system is given by the following expression

$$C=\sqrt{1-\sin^2 2yt\sin^2\theta}\,. \tag{11}$$

We note that the parameters of the system are time dependent. The Heisenberg spin-spin coupling and the DM coupling are assumed time-dependent and are defined as follows

1) $J=\frac{J_0}{\omega}\sin\omega t, D=\frac{D_0}{\omega}\sin\omega t,$

2) $J=\frac{J_0}{\omega}\sin\omega t, D=\frac{D_0}{\omega}\cos\omega t.$ (12)

Since these parameters depend on time-varying parameters such as the environment temperature that may change with time, their time-dependent behavior is important. In fact, the time-dependence of the systems' interaction describes the best the reality of the dynamics of the system.

3. Results and discussion

In the present paper, we focus our calculations and analysis on the time-dependency of the system's interaction. We have used MATLAB coding to calculate the numerical results. For better understanding, first we study the entanglement dynamics of the system with constant Heisenberg and DM interactions as observed in Fig. 1. It is obvious that the system with XXX Heisenberg interaction possesses the maximum entanglement (C=1) over the time. The presence of the DM interaction causes an oscillatory behavior in the entanglement dynamics. Raising the ratio of D/J increases the fluctuations of the correlation between two qubits. This implies the negative role of the DM interaction for entanglement dynamics of a two-qubit Heisenberg model. As a matter of fact, the rotational effect of DM interaction on spins, may destroy the arrangement of the XXX entangled model.

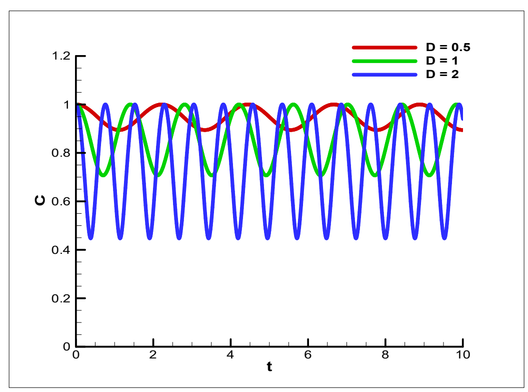
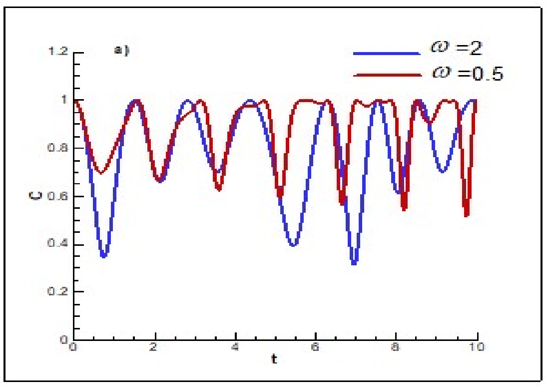


Figure 1. Time dependence of the concurrence for a two-qubit system with constant Heisenberg and DM interactions. Set J=1.

Now, we assume the Heisenberg interaction is time-dependent as a sinusoidal function. DM interaction is considered constant: $J=J_0\sin\omega t$, $D=D_0$. As observed in Fig. 2, compared to a system with constant strength of the Heisenberg interaction (Fig. 1), the amplitude of the oscillations increases, and the correlation experiences the irregular fluctuations.

Although, in higher angular frequency, the fluctuations become more regular, the time-dependency of the Heisenberg interaction causes the system to bear more severe fluctuations. Therefore, in order to obtain a more stable entanglement, it is better to keep the Heisenberg interaction constant over time or reduce its oscillation frequency. Figure 2b shows that the DM interaction intensifies the oscillating behavior of the fluctuations. In fact, in systems with time-varying Heisenberg interaction, weak DM interaction can help the stability of the system's entanglement over time. In the absence of the DM interaction, concurrence always remains in a constant value of 1, whether Heisenberg interaction is considered constant or time-dependent.



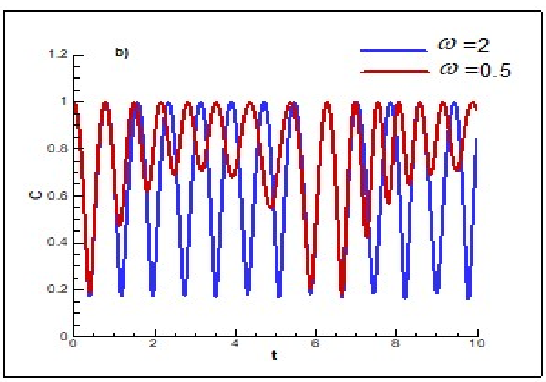


Figure 2. Concurrence as a function of time for time-dependent Heisenberg model: $J = J_0 \sin \omega t$, $D = D_0$. a)D_0=1, b) D_0=2. Set J_0=1.

If the amplitude of the sinusoidal behavior of Heisenberg interactions is also time dependent, namely $J = \frac{J_0}{\omega} \sin \omega t$,as observed in Fig. 3, reducing the angular frequency of the Heisenberg interaction improves the stability of the concurrence over the time. Compared to Fig. 2, we observe that in case of both amplitude and phase of the Heisenberg interaction are ω-dependence, the fluctuations of the entanglement dynamics reduce. Especially for smaller values of ω, the concurrence will gain more stability over time.

Up to here, aiming for a stable entanglement in the system, it would be better to choose a time-dependent Heisenberg interaction as $J = \frac{J_0}{\omega} \sin \omega t$ with low angular frequency as well as weak DM interaction.

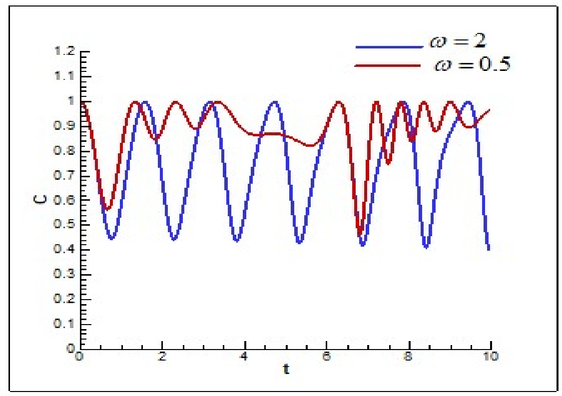


Figure 3. Concurrence as a function of time for time-dependent Heisenberg model: $J = \frac{J_0}{\omega} \sin \omega t$, $D = D_0$. Set J_0=1, D_0=1.

Finally, we study the model in which both J and D parameters are time- dependent as follows

$J = \frac{J_0}{\omega} \sin \omega t$, $D = \frac{D_0}{\omega} \sin \omega t$, in this case, the dynamics of the entanglement presents an interesting behavior. Since the effect of J and D parameters on the entanglement dynamics is opposite to each other, the effect of DM interaction can be relatively neutralized as observed in Fig. 4. In this case, the entanglement achieves a more stable status. A more comprehensive view can be seen in the 3D figure (Fig. 5). Figure 5 illustrates the concurrence in terms of time and angular frequency variations. The oscillatory behavior of concurrence becomes more intense by rising ω. However, the oscillation's amplitude doesn't change much.

Using Fig. 5, we can see the periodic behavior of entanglement over time. Therefore, in order to get the maximum information from the entangled system, it is enough to measure the system at the right times.

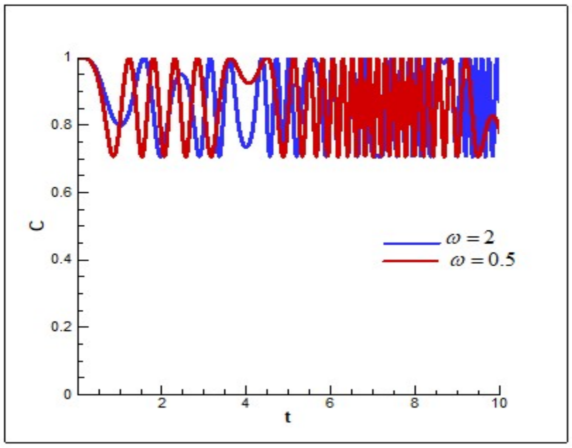


Figure 4. Concurrence as a function of time for time-dependent Heisenberg model: $J = \frac{J_0}{\omega}\sin\omega t$, $D = \frac{D_0}{\omega}\sin\omega t$. Set J_0=1, D_0=1.

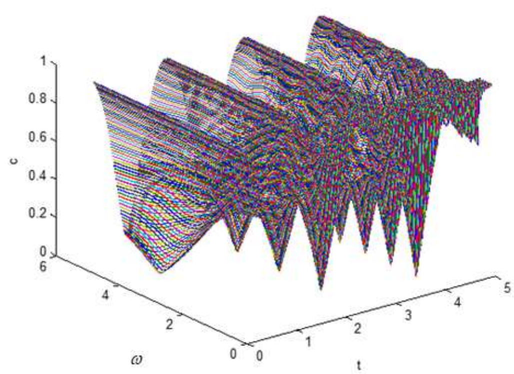


Figure 5. 3D scheme of concurrence as a function of time for $J = \frac{J_0}{\omega}\sin\omega t$, $D = \frac{D_0}{\omega}\sin\omega t$. Set J_0=1, D_0=1.

What happens if the time-dependency of the DM interaction is proportional to the cosine function? Figure 6 compares the dynamic behavior of systems with $D \propto \sin(\omega t)$ and $D \propto \cos(\omega t)$. As depicted in figure, the time-dependency as cosine function causes the strong fluctuations of the entanglement even sometimes the entanglement reaches zero. It may be due to the phase opposition of the Heisenberg interaction and the DM interaction $(DM \propto \cos(\omega t).J \propto \sin(\omega t))$. This finding can be important for designing the time-dependency of the DM interaction and Heisenberg interaction. In fact, if time-dependency of the both interaction is same, the dynamic behavior of the quantum correlation becomes more stable and the temporal fluctuations reduce significantly.

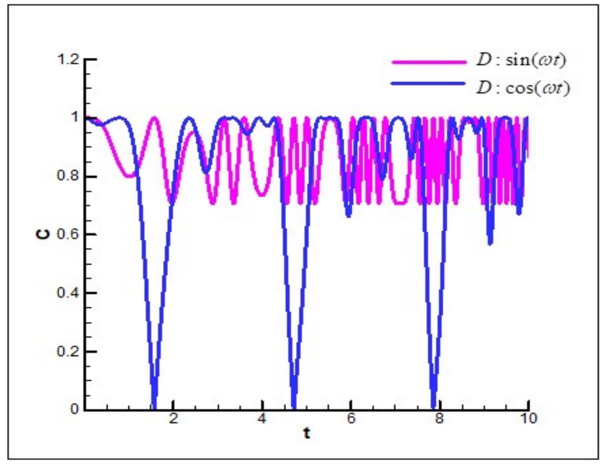


Figure 6. concurrence as a function of time for time-dependent Heisenberg model: $J = \frac{J_0}{\omega}\sin\omega t$, $D = \frac{D_0}{\omega}\cos\omega t$. Set J_0=1, D_0=1, $\omega = 2$.

4. Conclusions

In the present paper, we study the dynamic behavior of the quantum correlation in a two-qubit system with XXX Heisenberg model added by DM interaction.

Although, the two-qubit XXX Heisenberg model is a fully entangled system, adding the DM interaction causes some fluctuations in the dynamic behavior of the system's entanglement.

To make the system more realistic, the Heisenberg interaction and DM interaction are considered as a time-dependent function. If only the strength of the Heisenberg interaction is a time-varying parameter as $J = J_0\sin\omega t$, the amplitude of entanglement's fluctuations increases. This is attenuated by increasing the angular frequency of ω. If time-dependency of the Heisenberg strength is defined as $J = \frac{J_0}{\omega}\sin\omega t$, reducing the angular frequency of Heisenberg interaction improves the stability of the concurrence.

In case where both J and DM parameters are time-dependent as a sinusoidal function, namely $J = \frac{J_0}{\omega}\sin\omega t$, $D = \frac{D_0}{\omega}\sin\omega t$, the dynamics of the entanglement presents an interesting behavior. Since the effect of J and D parameters on the entanglement dynamics is opposite to each other, the effect of DM interaction can be relatively neutralized. In this case, the entanglement achieves a more stable status. The final case which is considered in the paper, is the time-dependency of the DM interaction as $D \propto \cos(\omega t)$. The findings imply that this kind of time-dependency causes the strong fluctuations of the entanglement so that the entanglement reaches zero in sometimes. Since the time-dependent interactions of a spin system have been investigated in a few studies, our study investigates the effect of the simultaneous interactions of Heisenberg and Dzyaloshinskii-Moriya on the dynamic behavior of the concurrence, and we have found noticeable results which could be the beginning of a new path for the study of quantum systems with time-dependent interactions.

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