Journal of Interfaces, Thin films, and Low dimensional systems 5 (1) Summer and Autumn (2021) 435-443

Light hypernuclei formation in the hyperonization process of quark-gluon plasma

Scientific research paper

Arezu Jahanshir*

Department of Physics and Engineering Sciences, Buein Zahra Technical University, Iran

ARTICLE INFO

Article history:
Received 9 March 2022
Revised 29 June 2022
Accepted 30 June 2022
Available online 3 July 2022

Keywords
Hyperonization
Bound states
Hadronic molecule
Schrödinger equation

ABSTRACT

The purpose of this article is to describe the relativistic corrections to the spectrum of bound states during hyperonization when interactions between quarks and gluons in semiquark-gluon plasma occur before experiencing color decay and color transformation between particles. I will consider this issue, according to the asymptotic behavior of the loop function in the scalar external gauge field based on the quantum field. This opinion was formed and presented using the projective unitary representation method and technique (oscillator representation method) based on the Schrödinger equations converge toward the semi-relativistic equation which can take into account some relativistic effects of mass and interaction in a coupled system. Such calculations represent the interaction between the hyperon and the nuclei core when the quark-gluon plasma cools down and we cannot see the free quarks and gluons, it occurs near the 150 MeV temperature in the quark-gluon plasma environment. The constituent mass and mass spectrum of hypernuclei are presented with relativistic corrections. It is a new calculation and description of a coupled state of hadrons based on quantum field theory and relativistic effect of interactions.

1 Introduction

One of the main subjects of high energy physics is the investigation and study of hyperon and hypernuclei production in different experiments and measurements in the Relativistic Heavy Ion Collider. In this paper, results of the bound state investigation of light hypernuclei in high energy collisions and semi-quark-gluon plasma are presented. The highest exotic states of hadronic matter are one of the important missions of relativistic and ultra-relativistic physics. In the high-density, extremely hot quark-gluon plasma or very hot hadronic matter as we know can form exotic hadrons. Heavy hadrons are created by quarks in the quark-gluon plasma. Hypernuclei, consisting of hyperon and nucleon are a very important subject in nuclear physics and nuclear astrophysics. The exotic atomic and molecular bound states contain those which do not fit in the normal well-known hadronic states. So, they include the muonic atom, kaonic atom, multiquark states, hypernuclear states, and hyperatom states [1-5]. The Λ-hypernuclear system is a nuclear two-component hadronic cluster that has previously been investigated in relativistic and nonrelativistic interactions with different potential models and methods such as quark-meson coupling model, microscopic cluster model [4], MIT Bag model, and the Gaussian expansion method [6-7]. But there exists a method towards understanding

*Corresponding author.
Email address: Jahanshir@bzte.ac.ir
DOI: 10.22051/jitl.2022.39787.1070

the hypernuclei states as di-hadron molecules reached from the approximate solution or perturbative methods in QCD and hadronic physics. It is widely believed that the oscillator representation method and quantum field theory is capable to extract the properties and specifics of hypernuclei systems with charged or non-charged hyperon, however, there exist theoretical and computational works on unusual nuclear-restricted states, for the most of their mass, eigenenergy, excited states, wave function [6-11].

Recently, light hypernuclei and hypernuclei clusters were produced as a hadronic-exotic bound state in the experimental cross-examination of high-energy hadronhadron collisions. These experimental observations on hypernuclei have been collected by SKS, FINUDA Collaboration, DAΦNE machine, PANDA Experiment, KEK, and by a new facility in Japan, J-PARC are expected that a higher aspect and new research fields will be given to hyper physics like the research and investigation on unusual hadronic systems properties and characteristics, which may be possible at new machines with higher energies. Therefore, the theoretical studies lead to greater awareness and interest in experimental analysis and interpretations of hypernuclei and hyperon-hadron interactions. Looking for the latest experimental data on hypernuclei, we study Λ-hypernuclear as an exotic molecule bound system like a di-hadronic molecule: hyperon-core (Λ- N). For the binding energy of the di-hadronic state of Λ-hypernuclei, we suggest using the asymptotic properties of Gaussian processes of the comparability and community correspondence currents in the field for the determination of the eigenvalue and binding mass in the ground and excited state of the di-hadronic molecules two-body system Λ-N with pseudoharmonic potential interaction. We also get a relativistic modification and adjustment to the mass of building blocks of the nuclear core (with $me \approx 0$) and Λ- hyperon. The mass spectrum is determined from the Schrödinger equation with a mass of constituent components neutral N and Λ-hyperon.

The exotic atomic and molecular states include those which do not fit in the normal well-known states. So, they include the muonic atom, kaonic atom, multiquark states, hypernuclear states, and hyperatom states [1-3]. The Λ-hypernuclear system is a nuclear many-body state that has previously been studied in the nonrelativistic potential models and framework of different models and methods such as quark-meson coupling model, microscopic cluster model [4], MIT Bag model, and the Gaussian expansion method [5]. But there exists a method towards understanding the hypernuclei states as di-hadron molecules based on the perturbative methods in QCD at the hadronic scale. We use the oscillator representation method (ORM) and quantum field theory to determine the characteristics of the di-hadronic molecule system of Λ-N with potential interaction. These exotic states are two-cluster bound states. The mass spectrum and constituent mass of particles in hypernuclei using the molecular pseudoharmonic-type potential between core and hyperon are investigated. The eigenenergy of hypernuclei is presented and defined from the Schrödinger equation for the bound state of constituent components hadronic nuclear N and Λ-hyperon.

2 Theoretical framework

Presently a set of experimental abilities exists. Let us study the properties of exotic hadrons consisting of light quarks based on the relativistic character of interaction. We know that the exposition of properties of exotic coupled states in an experimental model or quark-gluon plasma (QGP) environment with a strong constant of interaction is only a relativistic effect and is solved only within the framework of quantum field theory (QFT). Quantum field theory is one of the alternative methods for determining the bound state mass based on the nonperturbative and relativistic behavior of high-energy interaction. The issue of defining the mass of the bound state is an important problem that arises in describing the high-energy interaction. The mass spectrum of the particles as a bound state can be determined within QFT due to particle charge. Therefore, we presented the interaction of two charged scalar particles in a well known external field $A\alpha(x)$ and consider that the system of these particles creates a bound state and stable state. Based on QFT the mass of the coupled states can be described by the asymptotical action of the correlator, the polarization loop (close two-point loop), for these particles in our gauge field. The scalar charged particles current is, for defining the coupled mass based on correlator (polarization loop function) which can be described by the Green's function, all annihilation channels are neglected and Green's function product of scalar particles with masses in the external field can be presented by averaging over the field $A_\alpha(x)$ and restrict results to the lowest order terms, i.e. a two-point Gaussian correlation

$$\left\langle \exp\left\{ i\int dx A_\alpha(x) J_\alpha(x)\right\}\right\rangle_A = \exp\left\{-\frac{1}{2}\int\int dx dy J_\alpha(x) D_{\alpha\beta}(x - y) J_\beta(y)\right\}. \quad (1)$$

Here $J_\alpha(x)$ is the actual current. The propagator of a gauge field has a form

$$D_{\alpha\beta}(x-y) = \langle A_\alpha(x) A_\beta(y)\rangle_A.$$

The polarization loop function of a scalar particles in an external field $A_\alpha(x)$ reads

$$\Pi(x) = \langle G_{m_1}(x_1, x_2|E) \cdot G^*_{m_2}(x_2, x_1|E)\rangle, \quad (2)$$

and as we know, the Green's function $G_m(x_1, x_2|A_\alpha(x))$ is determined by

$$\left[\left(i\frac{\partial}{\partial x_a} + \frac{g}{c\hbar}A_a(x)\right)^2 + \frac{c^2m^2}{\hbar^2}\right]G(x,y|A) = \delta(x-y), \quad (3)$$

where m is the mass of a scalar particle, and g is the coupling constant of interaction. For determination of the loop function, first of all, the solution of Eq. (3) isrepresented as a functional integral [11]:

$$G(x,y|A) = \int_0^\infty \frac{ds}{(4s\pi)^2}\exp\left\{-sm^2 - \frac{(x-y)^2}{4s}\right\} \times \int d\sigma_\beta \exp\left\{ig\int_0^1 d\xi \frac{\partial Z_\alpha(\xi)}{\partial \xi}A_\alpha(\xi)\right\}, \quad (4)$$

where

$$Z_\alpha(\xi) = (x-y)_\alpha + y_\alpha\xi - 2\sqrt{s}B_\alpha(\xi),$$
$$d\sigma_\beta = N\delta\vec{B}\exp\left\{-{}^1/_2\int_0^1 d\xi\, \dot{\vec{B}}^2(\xi)\right\}, \quad (5)$$

with a normalization [11]

$$B_\alpha(0) = B_\alpha(1); \quad \int d\sigma_\beta = 1. \quad (6)$$

Substituting Eq. (6) in Eq. (2) and carrying out an averaging on the field $A_\alpha(x)$ for the polarization function with two-point closed loops, one can define

$$\Pi(x) = \int_0^\infty\int_0^\infty \frac{d\mu_1 d\mu_2}{(8x\pi^2)^2}\exp\left\{-\frac{1}{2}|x|\left(\frac{m_1^2}{\mu_1} + \mu_1\right) - \frac{|x|}{2}\left(\frac{m_2^2}{\mu_2} + \mu_2\right)\right\}J(\mu_1,\mu_2), \quad (7)$$

where

$$f(\mu_1,\mu_2) = N_1N_2\,\delta\vec{r}_1\delta\vec{r}_2\exp\left\{-\frac{1}{2}\int_0^x d\tau\left(\mu_1\dot{\vec{r}}^2{}_1(\tau) + \mu_2\dot{\vec{r}}^2{}_2(\tau)\right)\right\}\exp\{-W_{1,1} + 2W_{1,2} - W_{2,2}\}, \quad (8)$$

here $W_{i,j}$ is a potential for interaction and reads [11]

$$W_{i,j} = \frac{g^2}{2}(-1)^{i+j}\int_0^x\int_0^x d\tau_1 d\tau_2\, \dot{Z}_\alpha^{(i)}(\tau_1), D_{\alpha\beta}\left(Z^{(i)}(\tau_1) - Z^{(j)}(\tau_2)\right)\dot{Z}_\beta^{(j)}(\tau_2). \quad (9)$$

Then the mass of a coupled system is determined through the natural logarithm of the polarization function

$$M = -\lim_{|x-y|\to\infty}\frac{ln\,\Pi\,(x-y)}{|x-y|}. \quad (10)$$

The functional integral form of the polarization function and Green's function can describe all the above equations similar to the Feynman path integral in unrelativistic quantum mechanics for acting two particles with masses μ_1, μ_2 and interactions between these particles are described by $W_{i,j}$, which contain both potential, and un-potential interaction parts. Pay attention to the limited behavior of $|x-y| \to \infty$ from Eq. (10), the mass of a coupled system is determined as

$$M = \sqrt{m_1^2 - 2\mu^2 \frac{\partial E(\mu)}{\partial \mu}} + \sqrt{m_2^2 - 2\mu^2 \frac{\partial E(\mu)}{\partial \mu}} + \mu \frac{\partial E(\mu)}{\partial \mu} + E(\mu), \qquad (11)$$

where the parameter μ is the reduced mass and determined as

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} = \frac{1}{\sqrt{m_1^2 - 2\mu^2 \frac{\partial E(\mu)}{\partial \mu}}} + \frac{1}{\sqrt{m_2^2 - 2\mu^2 \frac{\partial E(\mu)}{\partial \mu}}}. \qquad (12)$$

Parameters μ_1, μ_2 are the component's mass of the coupled system and m_1, m_2 are the rest masses. $E(\mu_1, \mu_2) = E(\mu)$ is the eigenvalue of the nonrelativistic Hamiltonian defined by

$$\lim_{|x|\to\infty} J(\mu_1, \mu_2) = const. \exp\left(-|x| E(\mu_1 \mu_2)\right). \qquad (13)$$

3 Λ-hypernuclei as molecular states

The Λ-hypernuclei as a di-hadronic molecular system consisting of ordinary Λ-N are studied. According to the idea of quantum theory of scalar fields and the oscillator representation method, the mass spectrum is determined. Similar to this concept, solution of the hydrogen atom mass spectrum problem was discussed by Fock via variables representing and transformation into a four-dimensional momentum space, for details see [6]. We know that the quantum harmonic oscillator Hamiltonian reads4

$$\hat{H} \cong \frac{\hat{p}^2}{2\mu} + \frac{\mu}{2}\omega_0^2 r^2 - E_{0n}(\mu). \qquad (14)$$

Here, the Λ-hyper molecule Hamiltonian is based on the semi-relativistic Schrödinger equation in the pseudoharmonic potential between the two hadrons Λ-N assumed as

$$\hat{H}\Psi = E_n \Psi \quad \Rightarrow$$

$$\hat{H}\Psi = \left(M - \frac{1}{2}(\mu_1 + \mu_2) - \frac{1}{2}\frac{m_1\mu_2 + m_2\mu_1}{\mu_1\mu_2}\right)\Psi$$

$$\hat{H} = \frac{\hat{p}^2}{2\mu} + V_0\left(\frac{r}{r_0} - \frac{r_0}{r}\right)^2 - E_{0n}(\mu), \qquad (15)$$

where M is the mass of hyper-molecule bound state, m_1, m_2 are the masses of Λ-N. μ_1, μ_2 are the constituent masses of Λ-hyperon and core N. μ is the reduced mass, $E_{0n}(\mu)$ is the excited eigenenergy of n- state in the first approximation of ORM, r_0 is the equilibrium intermolecular Λ-N separation, which could be calculated by the empirical formula $r_0 = 0.59 + 0.83A^{1/3}$ in Eq. (15) and it can be defined by experimental data, V_0 is the dissociation energy between Λ-N or the absolute value of the binding energy B_Λ of a hypernuclear system [7].

In Eq. (14) we have not added the spin-orbit interactions. After variables representing $r = q^{2\xi}$ and transformation into axillary d-dimensional space in ORM, $\xi = \frac{d-2}{2(1+4\ell)} = 1/2$, then Eq. (14) reads:

$$\hat{H}_q = \frac{\hat{p}_q^2}{2} + 4\mu\xi^2\hat{q}^{4\rho-2}\left(\frac{V_0}{{q_0}^2}\hat{q}^2 + \frac{{q_0}^2}{V_0}\hat{q}^{-2} - 2V_0 - E_{0n}(\mu)\right) = H_0 + \varepsilon_0(E,\mu) + :H_I: \qquad (16)$$

where $\hat{q}, \hat{p}_q$ are canonical operators and can be presented by $\hat{a}^+$ and $\hat{a}^-$ operators:

$$\hat{a}^- = \sum_k \langle q|p\rangle \hat{a}^- = 2^{-1/2}(\hat{Q} + i\hat{P}_q),$$

$$\hat{a}^+ = \sum_k \hat{a}^+\langle p|q\rangle = 2^{-1/2}(\hat{Q} - i\hat{P}_q), \qquad (16*)$$

where

$$\hat{q} = \left[\frac{m\omega_0}{\hbar}\right]^{1/2}\hat{Q} = \left[\frac{2m\omega_0}{\hbar}\right]^{1/2}(\hat{a}^+ + \hat{a}\,),$$

$$\hat{p}_q = [\hbar m\omega_0]^{1/2}\hat{P}_q = i\left[\frac{m\omega_0}{2\hbar}\right]^{1/2}(\hat{a}^+ - \hat{a}\,).$$

The canonical variables as Wick ordering condition are obtained in Eq. (16), and then

$$r = q^{2\rho}, \quad \hat{q} = \frac{\hat{a}^- + \hat{a}^+}{\sqrt{2\omega_0}}, \qquad \hat{q}^2 \cong \frac{d}{2\omega\omega_0},$$

$$\hat{p} = \sqrt{2\omega_0}\frac{\hat{a}^- - \hat{a}^+}{2i}, \quad \hat{p}^2 \cong \frac{d}{2}\omega_0$$

here, $\hat{p}_q^2$ is the relative momentum of Λ-N, $H_0 = \omega_0(\hat{a}^+\hat{a}^-)$ is the energy of the free oscillator, $:H_I:$ is the interaction Hamiltonian, $\varepsilon_0(E,\mu)$ is the ground state energy of the bound state in ORM and is the higher level of variational approximation for the vacuum energy of the Hamiltonian. In this article ground and excited states are described by the full Hamiltonian in the normal form, which does not contain any perturbation order and terms with the order of $q^{2m}, m < 1$, i.e., $:H_I: \approx 0$ in ORM [6]. Then, the energy of the ground $n = 0$ and $n > 0$ excited state in the zeroth approximation of the ORM, is obtained by minimizing the expectation value of Hamiltonian:

$$\varepsilon_0(E,\mu) = \frac{\hat{p}_q^2}{2} + \frac{V_0}{q_0^2}\mu\hat{q}^2 - 2\mu V_0 - \mu E_{0n}(\mu) = 0 \Rightarrow$$

$$\varepsilon_0(E,\mu) = \frac{(1.5+\ell)\omega_0}{2} + \mu\frac{V_0}{q_0^2}\frac{(1.5+\ell)}{\omega_0} - 2\mu V_0 - \mu E_{0n}(\mu) = 0, \qquad (17)$$

where $\varepsilon_0(E,\mu)$ is the free oscillator Hamiltonian or the minimum energy of the exotic molecular bound state (i.e., is the vacuum energy of the Hamiltonian and equivalent to the ground state energy).

We can formulate $\varepsilon_0(E,\mu) = 0$, $\frac{\partial\varepsilon_0(E,\mu)}{\partial\omega_0} = 0$, and consider all quadratic terms completely included in the free oscillator. Therefore, the energy eigenvalues and the oscillator free frequency in the ORM read:

$$E_{0n}(\mu) = \frac{(1.5+\ell)\omega}{2\mu} + \frac{V_0}{q_0^2}\frac{(1.5+\ell)}{\omega} - 2V_0 = \frac{(1.5+\ell)\sqrt{2V_0}}{q_0}\mu^{-\frac{1}{2}} - 2V_0\,, \qquad (18)$$

$$\omega = \sqrt{\frac{2\mu V_0}{q_0^2}} = \mu\omega_0, \qquad (18*)$$

and the pseudoharmonic energy spectrum or the ground state energy in the zeroth perturbation order $j = 0$ in ORM [6] defines

$$E_{00}(\mu) = \frac{1.5\sqrt{2V_0}}{q_0}\mu^{-\frac{1}{2}} - 2V_0. \qquad (19)$$

Then, the mass spectrum of Λ-hypernuclear bound state in the zeroth approximation of ORM with recoil effect of nuclear core:

$$M = \mu_1 + \mu_2 + \mu\, \dot{E}_{0n}(\mu) + E_{0n}(\mu),$$

$$\mu_1 = \sqrt{m_1^2 - 2\mu^2 \dot{E}_{00}(\mu)},$$

$$\mu_2 = \sqrt{m_2^2 - 2\mu^2 \dot{E}_{00}(\mu)}\,,$$

$$\dot{E}_{00}(\mu) = \frac{\partial E_{00}(\mu)}{\partial \mu} = -\frac{(1.5+\ell)\sqrt{2V_0}}{2q_0}\mu^{-\frac{3}{2}}. \quad (20)$$

The parameters V_0, q_0 for combinations of Λ-N are by reference [11] and the parameter μ is the root of the equation

$$\mu^{-1} = \left(m_1^2 + \frac{(1.5+\ell)\sqrt{2V_0}}{q_0}\mu^{\frac{1}{2}}\right)^{-\frac{1}{2}} + \left(m_2^2 + \frac{(1.5+\ell)\sqrt{2V_0}}{q_0}\mu^{\frac{1}{2}}\right)^{-\frac{1}{2}}. \quad (21)$$

Here we calculate the mass spectrum, the binding energy of Λ-hypernuclei without recoil effect $\ell = 0$ and results present in Table 1 therefore, we could define

$$M = m_{core} + \mu_\Lambda + \frac{(1.5+\ell)}{2q_0}\sqrt{2V_0}\mu^{-\frac{1}{2}} - 2V_0, \quad (22)$$

$$E_{bin} = \mu + \frac{(1.5+\ell)\sqrt{2V_0}}{2q_0}\mu^{-\frac{1}{2}} - 2V_0, \quad (23)$$

$$\mu_\Lambda = \sqrt{m_\Lambda^2 + \frac{1.5\sqrt{2V_0}}{q_0}\mu^{1/2}}\,, \quad (24)$$

$$\omega_0 = \sqrt{\frac{2V_0}{q_0^2}}\mu^{-1/2}, \quad (25)$$

and then calculate the numerical values of parameters of the Λ-hypernuclei in the ground and excited states without recoil effect of nuclear core ($m_2 = m_{core} = \infty$).

Comparing the results of Tables 1, we see that the results obtained numerically and analytically in ORM concerning the ground state are in good agreement.

Table 1. Calculated mass and energy spectrum, the constituent mass of Λ-hyperon, binding energies for Λ-hypernuclei, and oscillator frequency in ground states (in MeV). Theoretical and experimental data are taken from Refs [12,13,16,17].

	M	μ_Λ	E_{00}	bin	M_{exp}	M_{theory}
$^{8}_{\Lambda}$	7675.141	1115.720	14.239	7.443	7653.2	-
$^{8}_{\Lambda}$	7674.283	1115.721	13.622	7.183	7642.52	7663.42
$^{8}_{\Lambda}Be$	7673.151	1115.722	13.606	7.225	7642.86	-
$^{9}_{\Lambda}$	8610.527	1115.723	16.980	8.904	8578.69	-
$^{9}_{\Lambda}Be$	8612.957	1115.719	15.349	7.092	8563.69	-
$^{10}_{\Lambda}Be$	9547.436	1115.723	18.139	9.520	-	9531.28
$^{10}_{\Lambda}B$	9546.744	1115.725	17.700	9.309	9500.15	-
$^{11}_{\Lambda}B$	10483.526	1115.727	20.397	10.685	10429.69	-
$^{12}_{\Lambda}B$	11420.373	1115.731	22.935	11.853	11356.91	-
$^{13}_{\Lambda}C$	12358.299	1115.739	23.274	12.180	12278.95	12323.93
$^{16}_{\Lambda}O$	15171.496	1115.743	26.301	13.778	-	-

The defined results for the mass of hypernuclei including $^{8}_{\Lambda}He$, $^{8}_{\Lambda}Li$, $^{9}_{\Lambda}Be$, and $^{10}_{\Lambda}B$ are 7674.283, 8612.957 and 9546.744 MeV respectively, while their experimental values given in [13] are: 7653.2, 7642.52, 8563.69, and 9500.15MeV. We also investigate the binding energy of $^{8}_{\Lambda}He$, $^{8}_{\Lambda}Li$, $^{10}_{\Lambda}B$, $^{11}_{\Lambda}B$, $^{13}_{\Lambda}C$, and $^{16}_{\Lambda}O$ as plotted in Figure 1 where the graph shows the mass number A versus binding energy of bound state. Our theoretical calculation has been compared with theoretical and experimental data [15]. It may be mentioned that the results show a good agreement between the experimental and theoretical results.

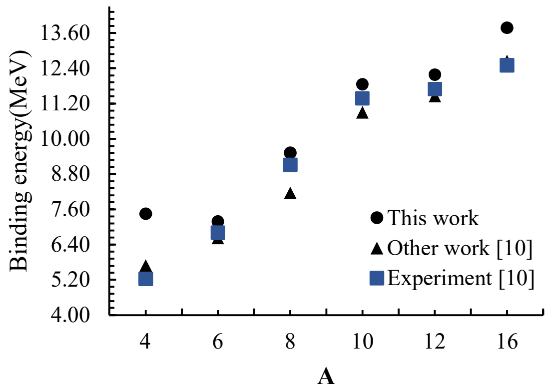

Figure 1. The mass number of hypernuclei versus binding energy.

4 Results

By considering the Λ-N states as di-hadronic molecules, their binding masses are computed. The high energy interactions of hadrons in quark-gluon plasma environments have been studied based on the stationary phase approximation characteristics of Gaussian processes of the comparability and community correspondence currents in the field. An analytic method for determining the mass spectrum and constituent masses of exotic hadronic bound states is presented. While the hadronic binding energy has been taken from the experimental data, the pseudoharmonic of di-hadronic molecular eigen energies have been determined using different approaches such as ORM. The ORM method is suggested for the determination of the mass spectrum of Λ-N states. The advantage of this procedure is its possibility to involve many different exotics-bound states.

Therefore, one has shown that the Λ-hypernuclei masses can be effectively evaluated from the yields of ORM and QFT methods with $\ell = 0$ and $\ell \neq 0$. The results demonstrated in Table 1 are identified and compared with other experimental data known as exotic hypernuclear atoms. These techniques and approaches can be applied for exotic multi hypernuclear systems, in which mass and eigenvalue were hard and difficult to evaluate and measure in previous hyper physics experiments. We believe such a kind of exotic multi hyperon nuclei would be possible at the new generation of ion accelerators of intermediate and high energies. One may be able to find many exotic di-hadronic states $J^{PC} = 0^{++}$ in these energies at the different sectors. Numerous hypernuclei have been identified with experimentally known exotic hadronic states. Among many combinations of hyperon-hadron nuclei and di-hadronic states like two-atom bound states which can be investigated, only a few exotics Λ-N state are described and introduced here in Table1.

Although the explanation and commentary on ${}^{A}_{\Lambda}Z$ is still doubted, I recognize it as Λ-${}^{A-1}_{Z}Z$ like two-atom bound states (molecular state). Many other di-hadronic modes were predicted such as ${}^{A}_{Y\acute{Y}}Z$, maybe experimentally identified. The theoretical masses of hypernuclei are to be determined. Some of them are compared with other theoretical works. Extensive research is going on to reveal the exact nature of exotic hypernuclei communication and connection. Therefore, the presented work describes hypernuclei as a multiplex complex quark state. The outcomes and obtained results are challenging and may throw light on our attempt towards understanding the fundamental main parts of new exotic matter, hypernuclei and the hyperon problem in neutron stars, long-lived hyper strange multiquark droplets, and strange quark matter. So, study of the properties of Λ-N state is a significant instrument for our understanding of the structure of exotic atoms and light Λ-N state, strange compact stars and exotic Λ-N interactions. Then, assuming the QFT, the numerical results based on calculation in this article for the $n = 1$ state binding energies, hypernuclei mass, and the constituent mass of Λ-particle considering Eqs. (9)–(12) are presented. The parameter μ including $\mu_1.\mu_2$ which can be determined by theoretical constituent masses for a rest mass of hypernuclei as seen from Eq. (7). To determine the main parameters of Λ-N state, we use the QFT and ORM ground state binding energies of Λ-N bound state. Therefore, we obtain a reduced mass. Considering the optimal reduced mass parameter and the potential parameter V_0 and q_0, we computed the theoretical analysis for the ground state of light Λ-N state and compared them with theoretical and experimental data taken from [13-17].

A good accordance between the experimental and theoretical data makes certain the reliability of the mass and eigenenergy extracted. In order to calculate the pure oscillator frequency and the eigenenergy of Λ-particle in the light hyperon-hadronic state, we considered two-body simple model Λ-hypernuclear as an exotic molecule bound systems like a di-hadronic molecule: hyperon-core (Λ-N), which is a utility model for those complications associated with the calculation of multiplex hadronic bound states. In Table 1, one can see, by increasing the mass number of hyperon-core (Λ-N) states, $({}^{8}_{\Lambda}Be, {}^{9}_{\Lambda}Be, {}^{10}_{\Lambda}Be)$ the value of the constituent mass of Λ-particle and the value of energy eigenvalue E_{00} of Λ-hypernucleus increases.

Also, by growing and increasing the proton number in the (Λ-N) states $({}^{8}_{\Lambda}He, {}^{8}_{\Lambda}Li, {}^{8}_{\Lambda}Be)$ the value of the constituent mass of Λ-hyperon increases. Note that, the characteristic of charge dependence of hyperon-core (Λ-N) interaction, i.e. proton-hyperon bound state interaction with more protons is stronger than with more neutrons. For example, the ${}^{8}_{\Lambda}Li$ is higher than the ${}^{8}_{\Lambda}He$ despite the electrostatic repulsion of protons which necessarily induces reduction energy eigenvalue of the ${}^{8}_{\Lambda}Li$.

The criterion for selecting these heavy exotic di-hadronic molecules is nothing but for their great importance in the hyper nuclear physics, strange stars, and neutron stars related areas, as we know ${}^{A}_{\Lambda}He$ and ${}^{A}_{\Lambda}Li$ are important di-hadronic molecules involved in many nuclear processes and some very essential topics in white dwarfs, neutron stars, and heavy-ion collision experiments.

5 Conclusions

Cadmium telluride nanoparticles synthesized by the sonochemical method in this work showed different structural properties when compared to nanoparticles produced by the previous methods. One of the most I present recent theoretical studies on heavy hyperon hypernuclei. I discuss the use of the Wick ordering method to determine the mass spectrum and make a comprehensive study of light (Λ-N) states in the ORM structure of interactions and potential models. The interaction model parameters, eigenenergy, and masses of the light (Λ-N) states obtained from the respective hyperon-core mass predictions have been used and applied to study exotic hypernuclei characteristics and specifications. As we know in the new generation facilities, such as J-PARC, MAMI, JLab, and FAIR very soon some new hypernuclei systems could be experimentally discovered. We have carried out analysis for the one-dimensional Schrödinger equation with a pseudoharmonic potential where the restrictions on the V0. q0 parameters have been given. The problem is then solved in an axillary d-dimensional space and the bound state energy solutions of exotic Λ-hypernuclei are found under the influence of ORM. The relativistic energy levels, mass spectrum, and constituent mass are obtained. As a further application, we have determined the pure oscillator frequency of a few exotic Λ-hypernuclei as di-hadronic molecules and developed an interest in these exotic molecules to be able to study better with the future research in the harmonic, molecular, Yukawa, and Coulombic potentials models.

References

[1] N. Buyukcizmeci, A. S. Botvina, R. Ogul, M. Bleicher, "Evaluation of hypernuclei in relativistic ion collision." The European Physical Journal A, **56** (2020) 210.

[2] C. Dover, A. Gal, "Hypernuclei." Annals of Physics, **146** (1983) 309.

[3] A. Jahanshir, "Strong interactions of hardon in quark cluster model." Scientific Journal of Pure and Applied Sciences, **4** (2015) 132.

[4] A. Jahanshir, "Di-mesonic molecules mass spectra." Indian Journal of Science and Technology, **10** (2017) 10.

[5] M. Tomaselli, T. Kuehl, and D. Ursescu, "Microscopic cluster model for exotic nuclei." Progress in Particle and Nuclear Physics, **59** (2006) 455.

[6] M. Thandar Aung, Th. Wint, K. S. Myint, et al., The European Physical J Web Conf. **206** (2019) 4.

[7] J. Hu, Y. Zhang, H. Shen. arXiv, (2021) 2104.13567v1.

[8] Zh. Feng. arXiv, (2021) 2109.01270.

[9] T. R. Saito, Dou, W. Drozd, et al., "New directions in hypernuclear physics." Nature Reviews Physics, **3** (2021) 803.

[10] E. Hiyama, "Gaussian expansion method for few-body systems nd its applications to atomic and nuclear physics." Progress of Theoretical and Experimental Physics, **1** (2012) 01A204.

[11] M. Dineykhan, G. V. Efimov, G. Ganbold, et al., Oscillator Representation in Quantum Physics, 2nd ed., Springer-Verlag, Berlin (1995) 210.

[12] N. Kolesnikov, V. S. Rostovskii, and M. N. Starosotnikov, "Formula for nuclear size taking into account shell effects." Physics Uspekhi, **31** (1986) 1131.

[13] J. Pniewski, "Hypernuclei and hypernuclear spectrospcopy." Acta Physica Polonica B, **2** (1971) 129.

[14] S. Pal, R. Ghosh, B. Chakrabarti and A. Bhattacharya, "A study on the ground and sxcited states of hypernuclei." Physica Scripta, **95** (2020) 045301.

[15] S. Pal, et al., "A study on binding energies of Λ hyernuclei." European Physical Journal Plus, **132** (2017) 262.

[16] P. A. Zyla, "Review of particle physics." Particle Data Group, Progress in Theoretical and Experimental Physics, **2020** (2020) 083C0.

[17] A. Armat and S. Mohammad Moosavi Nejad, "Ground state binding energies and RMS radii of Λ in hypernuclei in the presence of spin-dependent otential." International Journal of Modern Physics E., **28** (2019) 1950011.