Journal of Interfaces, Thin films, and Low dimensional systems 4 (1) Summer and Autumn (2020) 339-346

Spin-orbit interactions at finite temperature in low dimensional bound states

Scientific research paper

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ARTICLE INFO

Article history:
Received 15 July 2021
Revised 4 August 2021
Accepted 5 August 2021
Available online 26 October 2021

Keywords:
Bound state
Excitation
Relativistic mass
Spin interactions

ABSTRACT

Spin-orbit interactions of exciton relativistic bound states at a finite temperature in the framework of the projective unitary representation in a physics model with the Coulomb potential have been investigated. The ground state of the system in order to describe the temperature effect in a low dimension environment has been defined. The bound state, with electron-hole pair, has attracted a great deal of interest in thin-films and nanophysics. The reality of the state has been the subject of intense concern among theoreticians and experimenters in recent years. Spin-orbit interactions of exciton are considered to be in an electron-hole pair bound state. The problem of spin interactions of coupled states based on the quantum field theory in its widest sense is a method to control and achieve reasonable goals; and in this article, the problem is examined in details. The structure of the interaction Hamiltonian with the Coulomb type potential at finite temperatures is defined and then the mass and energy spectra of an exciton based on the spin interactions are determined theoretically. The defined properties at finite temperature can be used for new high technology materials of semi-conductive features for electronics, microelectronics, photovoltaic or solar cell manufacturing, and semiconductor chips.

1 Introduction

One of the well-known methods of considering interactions of the exciton state in low dimension thin films and materials was proposed in 1995, where the asymptotic of Green's function was obtained in the exponential form of propagators in an external field [1,2]. The method is applied to the basic fundamental model of quantum electrodynamics and quantum field theory. The polarization closed loop function through a functional integral is presented in this approach; this functional integral cannot be calculated as usual; in this case, we have to simplify physical assumptions as done in [1-4]. An alternative method of calculating the functional integral and determining the exciton properties at finite temperatures is defined in the quantum field theory. The finite temperature-dependent in a low dimension environment like thin films is a very important subject in theoretical and experimental physics.

Progress of exotic bound state physics in low dimensional transition semiconductor quantum dots allows exotic constructions, including poly-excitonic

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DOI: 10.22051/jitl.2021.36935.1057

systems, to be synthesized [1,2]. Specifically, a finite number of poly-excitons can be confined in a bounded volume of the order of normal-sized low-dimensional materials at a finite temperature. Therefore, the poly-excitonic environments have recently gained attention in theoretical investigations and experimental explorations. Theoretical physicists are interested in the possibility of controlling the properties of these materials at finite temperatures that could be implemented in new high technology materials of semi-conductive and conductive features for electronics, microelectronics, fuel cell production, photovoltaic or solar cell manufacturing, and semiconductor chips. The electron-hole state is the simplest model of bounding states to use when studying the essential features of ploy excitonic systems. Recently, the study of excitons has progressed significantly.

Among poly excitonic systems in low dimensional thin films, spin-orbit interactions of two exciton pairs are especially interesting because of their interaction effects in bound states. Many new poly excitonic bound states have been recently discovered following developments at finite temperatures and higher bandgaps making poly excitonic states to become the main topic of studies on low dimension materials and thin films. Many theoretical works have focused on determining or predicting the relativistic properties and conditions of poly excitonic states.

The present article investigates the asymptotic behavior of the correlation functions of charged fields and the analytic method for determining properties of poly excitonic states in a low dimension material at a finite temperature. According to the results, in the poly excitonic system, the total mass of the bound state differs from hole and electron masses in a free state. This work represents a theoretical and analytical effort where its outcome provides fundamental quantities of the poly excitonic electron-hole bound states. This outcome leads to make several predictions of the excitons in low dimension thin films and semiconductor quantum dots at a finite temperature. We choose to use the strong Coulomb interaction at a finite temperature as it plays a significant role in thin-film physics. The Schrodinger equation solutions for the Coulomb potential are known and can be obtained using various methods. In this article, we calculate the effect of the spin-orbit interaction in the bound state of poly excitonic systems in the ground state. Various analytical or numerical approximation methods have been developed to compensate for the fact that the relativistic Schrodinger equation for such a system does not produce solutions. In this way, one can demonstrate the projective unitary representation in physics [3,4] when calculating the properties of poly excitonic states. This technique can be used to accurately describe the characteristics of poly excitonic systems in thin films at finite temperatures. Thus, it is essential to develop the projective unitary representation in low dimension physics, as it describes the bound-state characteristics of systems such as exciton, diexciton, and poly excitonic systems.

2 Materials and methods

2.1 Mass spectrum in the relativistic limit

To define spin-orbit interactions in the poly-excitonic bound states at low dimensional materials at a finite temperature based on quantum electrodynamic filed and photon interaction wave function between electron-hole, we have to describe the main formula which contains the mass and energy eigenvalues. The temperature relation for the one-photon exchange (Coulomb potential) describes the new characteristic of the poly excitonic bound state in low dimension thin films and materials [2,5,6].

We try to describe interaction in temperature-dependent conditions by modified radial Schrödinger equation and using the analytic method based on the behavior of the correlation function of a photon at a finite temperature in the electromagnetic field. The determination of mass spectrum of poly excitonic systems is suggested within this idea where the binding energy and mass of the bound states are determined. The exciton-exciton, three excitons, etc. states include those which fit in the well-known states. They include in the multi-electron-hole states.

The exciton bound system is an electron-hole state that has been studied in the electrostatic field and framework methods such as the Gaussian expansion method, the quantum electrodynamics sum rules, and the Lattice quantum dynamics. Therefore, based on the quantum electrodynamic models and quantum field theory we can determine that the electron-hole system is one of the most crucial states which makes it possible to form

bound states of gauge bosons at low/high finite temperatures.

In this article, we show that the mass spectrum of the exciton state is extremely higher than predicted in theory at a finite temperature. We study exciton spin-orbit interactions at a finite temperature using the Gaussian asymptotic [2,3] behavior of the correlation functions of the corresponding field currents for the determination of the energy and mass spectrum in the ground state with the spin interactions in the Coulomb potential at a finite temperature. The mass spectrum and the constituent mass of the electron-hole system are determined from the modified Schrödinger equation for Hamiltonian of the Schrödinger equation [5,6]

$$\widehat{H}R(r) = MR(r) = E_0(\mu)R(r). \quad (1)$$

Now we describe the method: as we know, in the quantum field, the mass of the coupled particles is presented by the Gaussian shape of the correlated-current function and the exact orbital quantum numbers. The statistical correlation, in terms of the Green function $G(r)$, is expressed. It is defined as a functional integral and allows the necessary Gaussian limit to be allocated before one can carry out the average value of the external gauge field.

In nonrelativistic quantum mechanics, the resulting image of presentation is similar to the Feynman functional path integral. Hence, the mass of coupled electron-hole is determined by the polarization function (statistical correlation)

$\Pi(r) = \left\langle G_e(r|A)G_h^*(r|A)\right\rangle_A$, and the Green's function $G(r)$ [5]. We know, the current of scalar charged particles is $J(r) = R^+(r)R^-(r)$ where it is convenient to represent the considered correlators as the average over the gauge field $A(r)$ of a product of the Green's functions of the scalar charged electron hole in the electromagnetic field. The Green's function of the scalar charged electron and hole is defined by the equation

$$\left[\left(i\frac{\partial}{\partial r} + \frac{g}{c\hbar}\right)^2 + \frac{c^2m^2}{\hbar^2}\right]G(r|A) = \delta(r). \quad (2)$$

Solution of the Green's function is described in the functional integral form (for full detail see [2])

$$G(r|A) = \int_0^\infty dt\, \exp\{-m^2\alpha\}T_\tau \times \exp\left\{-t\int_0^1 d\tau\left(i\frac{\partial}{\partial r(\tau)} + gA\left(r(\tau)\right)\right)^2\right\}\delta(r). \quad (3)$$

Also, we know the coupled gluon-gluon mass spectrum is explain in relativistic-quantum theory by the polarization function (for full detail see [2])

$$\Pi(x) = A\cdot J_\mu(\mu) = \int_0^\infty\int_0^\infty \frac{du_1 du}{(8r\pi)^2}\exp\left\{-r\left(\frac{m^2}{\mu} + \mu\right)\right\}J_\mu(\mu),$$

where

$$J(\mu) = = N_1N_2\iint \delta r_1\delta r_2\exp\left\{-\frac{1}{2}\int_0^r d\tau\left(\mu\dot{r}_1^2(\tau) + \mu\dot{r}_2^2(\tau)\right)\right\}\exp\{-W_{1,1} + W_{1,2} - W_{2,2}\}.$$

The functional integral $J(\mu)$ looks like the Feynman path integral in nonrelativistic quantum mechanics for four-dimensional motion of particles with reduced mass μ. The interaction of these particles is defined by the nonlocal functional $W_{i,j}$ which contains potential and nonpotential interactions. The asymptotic of the function $J_\mu(\mu)$ looks like $J_\mu(\mu)\sim e^{(-rE_\ell(\mu))}$. Thus, we obtain:

$$\Pi(r) \cong\Rightarrow \exp\left(-M\sqrt{\Pi(r)^2}\right),$$

$$M = -\lim_{|r|\to\infty}\frac{\ln\Pi(r)}{|r|}. \quad (4)$$

After simplifying the equation, the mass spectrum reads $(i = 1,2)$

$$M = \frac{\partial}{\partial\mu_i}\left(\left(\frac{\mu_2m_1^2 + \mu_1m_2^2}{2\mu_1\mu_2}\right) + \frac{\mu}{2} + E(\mu)\right). \quad (5)$$

Therefore, the electron-hole system with the rest mass ($m_1 = m_e, m_h = m_2$), the exciton bound state mass in the stationary state reads

$$M = \frac{\partial}{\partial \mu_1}\left(\frac{\mu_2 m_1^2 + \mu_1 m_2^2}{2\mu_1\mu_2} + \frac{\mu}{2} + E_\ell(\mu)\right),$$

$$M = \frac{\partial}{\partial \mu_2}\left(\frac{\mu_2 m_1^2 + \mu_1 m_2^2}{2\mu_1\mu_2} + \frac{\mu}{2} + E_\ell(\mu)\right),$$

$$\mu = \frac{\mu_1 + \mu_2}{\mu_1\mu_2}. \tag{6}$$

By using the Taylor approximation one can determine the parameters $i = 1,2$ as follows

$$\mu_i = \frac{\partial M}{\partial \mu_{1i}} = 0 \Rightarrow$$

$$\mu_{1=}\sqrt{m_1^2 + \mu^2\left(\frac{e^2}{\mathcal{D}\pi\varepsilon_r}\right)^2} \rightarrow$$

$$\mu_{e=}\sqrt{m_e^2 + \mu^2\left(\frac{e^2}{\mathcal{D}\pi\varepsilon_r}\right)^2} \rightarrow$$

$$\mu^*{}_{e=}\sqrt{m_e^{*2} + \mu^2\left(\frac{e^2}{\mathcal{D}\pi\varepsilon_r}\right)^2},$$

and

$$\mu_{2=}\sqrt{m_2^2 + \mu^2\left(\frac{e^2}{\mathcal{D}\pi\varepsilon_r}\right)^2} \rightarrow$$

$$\mu_{h=}\sqrt{m_h^2 + \mu^2\left(\frac{e^2}{\mathcal{D}\pi\varepsilon_r}\right)^2} \rightarrow$$

$$\mu^*{}_{h=}\sqrt{m_h^{*2} + \mu^2\left(\frac{e^2}{\mathcal{D}\pi\varepsilon_r}\right)^2}, \tag{7}$$

where m_e^*, m_h^* are the electron and hole effective mass, and m_e, m_h are the electron and hole rest mass. Based on the electron-hole interaction we use the effective mass, constituent mass, and reduced mass as expressed by $m_e^* = m_e, m_h^* = m_h$, in the following expressions

$$\mu = \mu^* = \frac{\mu^*{}_1 + \mu^*{}_2}{\mu^*{}_1\mu^*{}_2},$$

$$\mu^*{}_1 = \mu{}_1.\mu^*{}_2 = \mu{}_2,$$

and also from Eq. (4) we define

$$E_\ell(\mu) = M - \frac{1}{2}((\mu_e + \mu_h) - \frac{m_e^2\mu_h + m_h^2\mu_e}{2\mu_e\mu_h},$$

$$E'_\ell = \frac{dE_\ell}{d\mu}. \tag{8}$$

as we know that $J_\mu(\mu) \sim e^{(-rE_\ell(\mu))}$.

In Eq. (7) the particle's effective mass (denoted by m^*} is the mass that it seems to have when responding to forces, or the mass that it seems to have when interacting with other identical particles in a thermal distribution. One of the results from the bound state theory of exciton is that the movement of electron-hole in a potential field can be very different from their motion in a vacuum [7]. The effective mass is a quantity that is used to describe exciton's bound states by modeling the behavior of a free particle (electron-hole) with that mass. For some purposes and some nanomaterials, the effective mass can be considered to be a simple constant of a material while the value of effective mass depends on the purpose for which it is used, and can vary depending on several factors [7] especially in nano quantum dots. For electrons or electron holes in a solid, the effective mass is usually stated in units of the rest mass of an electron (9.11×10−31 kg). In these units, it is usually in the range 0.01 to 10, but can also be lower or higher—for example, reaching 1000 in exotic heavy fermion materials, or anywhere from zero to infinity (depending on definition) in graphene. As it simplifies the more general band theory, the electronic effective mass can be seen as an important basic parameter that influences measurable properties of a solid, including everything from the efficiency of a solar cell to the speed of an integrated circuit.

3 Schrödinger equation for the bound state

The radial Schrödinger equation for the multiplex system with Coulomb interaction is [5]:

$$H = H_0 + \hat{H}_{spin} =$$

$$\frac{1}{2}\sum_{i=1}^{n} m_i\, v_i^2 + \sum_{i=1}^{n-1} V_{i<j} + \hat{H}_{spin}. \qquad (9)$$

Now, for defining characteristics of the excitonic systems in the electromagnetic field interactions, we use the bound state in the electrical confining potential at the finite temperature $V(r,T) = -\frac{a(r,T)}{r}$ [7]. In this case, using the modified radial relativistic Schrödinger equation (Eq. (9)), we have

$$\hat{H}R(r) = MR(r) = E_\ell(\mu)R(r).$$

$$MR(r) = \left(\sqrt{m_e^2 + \hat{p}_r^2} + \sqrt{m_h^2 + \hat{p}_r^2} + \right.$$

Now, we explain the relativistic effects on the bound states using explanations *I* and *II*:

$$I.\ \sqrt{m^2 + \hat{p}_r^2} = m\sqrt{1 + \frac{\hat{p}_r^2}{m^2}} \approx$$

$$\approx m + \frac{\hat{p}_r^2}{2m} - \frac{\hat{p}_r^4}{8m^3} + \cdots,$$

$$II.\ \sqrt{m^2 + \hat{p}_r^2} \approx \min_{\mu}\left(\mu + \frac{m^2 + \hat{p}_r^2}{\mu}\right).$$

Two approximate methods are usually used for predicting the structure of the bound states before obtaining the Hamiltonian of the bound state as ($\hbar = c = 1$)

$$\left[\frac{1}{2}\left(\mu_1 + \frac{m_e^2 + \hat{p}_r^2}{\mu_e}\right) + \frac{1}{2}\left(\mu_2 + \frac{m_h^2 + \hat{p}_r^2}{\mu_h}\right) + \frac{a(r.T)}{r}\right]R(r) = MR(r). \qquad (11)$$

or

$$\left[\frac{\hat{p}_r^2}{2\mu} + \frac{a(r.T)}{r}\right]R(r) = \left[M - \frac{1}{2}(\mu_e + \mu_h) - \frac{m_e^2\mu_h + m_h^2\mu_e}{\mu_e\mu_h}\right]R(r), \qquad (12)$$

where μ is the reduced mass and $\mu_{e,h} is$ the constituent mass of particles in the bounding system. The n-dimensional modified Schrödinger equation could be presented using [8-10]

$$\frac{1}{2r^{n-1}}\frac{d}{dr}\left(r^{n-1}\frac{d}{dr}\right)R(r) - \frac{\ell(\ell+n-2)}{2r^2}R(r) + \mu V(r.T)R(r) + \mu\left(M - \frac{1}{2}(\mu_e + \mu_h) - \frac{m_e^2\mu_h + m_h^2\mu_e}{\mu_e\mu_h}\right)R(r) = 0. \qquad (13)$$

Spin-orbit interactions at finite temperature in low dimensional bound states can be presented in the potential types: Cornell, Coulombic, and strong interactions by the exponential relations [8]. The Coulomb-temperature potential is described by the exponential function $V(r,T) \approx -\frac{1}{r}\exp(-a(T)\mu(T)r)$ the reduced mass [7] at $T \neq 0$. Using the approximation of the exponential function

$$\exp(-a(T)\mu(T)r) = \sum_{n=0}^{\infty}\left(\frac{(-a(T)\mu(T)r)^n}{n!}\right),$$

we obtain

$$V(r,T) \approx -\frac{1}{r} + \alpha(T)\mu(T) - \beta(T)(\mu(T))^2 r + \ldots$$

Therefore, the Hamiltoni of the exciton system is as follows [11,5]

$$\left[\frac{-1}{2\mu}\left[\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{\hat{\ell}}{r^2}\right] + + \frac{(-1 + \alpha(T)\mu(T)r - \beta(T)(\mu(T)r)^2)}{r} + H_{spins}\right]R(r) = E_0(\mu)R(r). \qquad (14)$$

One can describe the exact temperature relation in the Coulomb potential by

$$V(r,T) = Be^{-\mu(T)r} =$$

$$= B\left(1 - \mu(T)r + \frac{(\mu(T)r)^2}{2}\right).$$

We modify the variables in the starting Schrödinger equation based on the projective unitary representation method in the quantum field theory for the ground state at a finite temperature with described spin-orbit interactions. This state is described as an infinite number of oscillators that keep their oscillating character in interactions.

To use quantum field methods, we have to change variables in Eq. (14) for the linear interaction terms of the Coulomb potential by replacing a new variable in the $\mathcal{D}$-dimensional axillary space. Now, based on the asymptotic properties $(r \to \infty, r \to 0)$ of Gaussian type $r = q^{2\rho}$ and $R(r) \to R(r) = q^{2\rho}R(q^2)$, where ρ is a parameter to be determined; in the charge potential such a modification is performed by $\rho \cong 1$ where the electron-hole wave function becomes an oscillator. Using the radial Laplacian operator in the n-dimension space one can define the radial Laplacian operator in the $\mathcal{D}$-dimensional ($\mathcal{D} = 2\rho + 2$) axillary space [2]:

$$\Delta_r = \frac{d^2}{dr^2} + \frac{n-1}{r}\frac{d}{dr} \to$$

$$\Delta_q = \frac{d^2}{dq^2} + \frac{\mathcal{D}-1}{q}\frac{d}{dq}.$$

where

$$\frac{d}{dr} = \frac{dq}{dr}\frac{d}{dq} = \left(\frac{d}{dr}\left(r^{\frac{1}{2\rho}}\right)\right)\frac{d}{dq} = \left(\frac{1}{2\rho}q^{1-2\rho}\right)\frac{d}{dq},$$

$$\frac{d^2}{dr^2} = \frac{d}{dr}\frac{d}{dr} = \left(\frac{1}{4\rho^2}q^{2-4\rho}\right) \times \left(\frac{d^2}{dq^2} + \frac{1-2\rho}{q}\frac{d}{dq}\right).$$

Here, the wave function should have the Gaussian type solution for large distances and we apply the projective unitary representation method variables from $R(r) \to q^{2\rho}R(q^2)$ and the transformation canonical variables as Wick ordering creation and annihilation operators (see [2] for more details) $\hat{q} = \frac{\hat{a}^- + \hat{a}^+}{\sqrt{2\omega_0}}$, $\hat{p}_q = \sqrt{\frac{\omega_0}{2}}\frac{\hat{a}^- - \hat{a}^+}{i}$, then determine

$$q^2 = \frac{\mathcal{D}}{2\omega_0}, \quad q^4 = \frac{\mathcal{D}(\mathcal{D}+2)}{4\omega_0^2}, \quad p^2 = \frac{\mathcal{D}\omega_0}{2}$$

and define the Hamiltonian (14). Equation (14) in a new auxiliary space is obtained

$$\varepsilon_0(E_\ell, T) = \frac{\hat{p}_q^2}{2}R(q^2) + 4\mu q^2 H_{spin}R(q^2) - 4\mu q^2 E_0 R(q^2) + (-4\mu + 4\alpha\mu^2 q^2 - 4\beta\mu^3 q^4)R(q^2) = 0. \quad (17)$$

Now, we can find the renormalization of the bound state parameters like wave function which allows us to introduce the zero approximation in the projective unitary representation method and then find the eigenvalue of the ground state energy $\varepsilon_0(E_0)$. Thus, Eq. (17) is written in the form

$$\varepsilon_0(E_0.T) = X(T.\omega_0) - E_0 Y(T.\omega_0) = 0,$$

or

$$\varepsilon_0(E_0.T) = \left(\frac{\mathcal{D}}{4}\omega_0 - 4\mu + \right.$$

$$+\frac{2\mu\mathcal{D}}{\omega_0}[3\nabla_r V(r)](\hat{L}\hat{S}) + \frac{2\mu\mathcal{D}}{\omega_0}[\Delta_r V(r)](\hat{S}_e\hat{S}_h)$$

$$\left. +\frac{2\alpha\mu^2\mathcal{D}}{\omega_0} - \frac{\beta\mu^3\mathcal{D}(\mathcal{D}+2)}{\omega_0^2}\right) - \frac{2\mu\mathcal{D}}{\omega_0}E_0 = 0, \quad (18)$$

where we suppose $\hat{S}_+ = \hat{S}_1 + \hat{S}_2$ $(\hat{S}_1 = S_e, \ \hat{S}_2 = S_h)$ for the sum of two particles' spin in the exciton system (spin-orbital and spin-spin equations) where we will have

$$H_{spin} = H_{LS} + H_{SS} \to$$

$$H_{LS} = \frac{2\mu\mathcal{D}}{\omega_0}[3\nabla_r V(r)](\hat{L}\hat{S}),$$

$$H_{LS} = \frac{2\mu\mathcal{D}}{\omega_0}[\Delta_r V(r)](\hat{S}_1\hat{S}_2),$$

$$j = \ell + S,$$

$$(\hat{S}_1\hat{S}_2) = \frac{\left(S(S+1) - S_1(S_1+1) - S_2(S_2+1)\right)}{2}, \qquad (19)$$

$$(\hat{L}\hat{S}) = \frac{\left(j(j+1) - S(S+1) - \ell(\ell+1)\right)}{2},$$

where based on the projective unitary representation conditions (Eq. (18)), we find

$$\varepsilon_0(E_0, T) = 0, \quad \frac{d\varepsilon_0(E_0,)}{d\omega_0} = 0, \qquad (20$$

)and the ground state energy$E_0 = E_0(\mu.T)$ and also the pure oscillator frequency of the exciton system in the ground state $\omega_0 = \omega_0(E_0, T)$ at a finite temperature. The pure oscillator frequency of the exciton system in the Coulomb potential without temperature relation reads $\omega_0 = \sqrt{-8\mu\, E(\mu)}$. Thus, using formulas (18, 20) and (7), then one can determine the mass spectrum of the predicted bound state. Based on Eq. (20), the electron-hole system as an exciton bound state solution by the radial modified Schrödinger equation at a finite temperature has been described. For this opinion, the projective unitary method of the Schrödinger equation is used. The behavior of the exciton bound state at a high temperature is very important in low dimension materials and thin-film environments. We have presented the bound state spin-orbit and relativistic mass spectrum based on quantum field theory and have determined the relationship between mass spectrum and temperature in the Coulomb type potential at a finite temperature. The temperature-dependent radial modified Schrödinger equation is investigated by applying the projective unitary method in the ground state wave function of the exciton bound system ($\ell = 0, \mathcal{D} = 3$). Now, we determine the finite temperature as described by [2,12] nonzero and zero temperature spin interactions and mass spectrum of the electron-hole. Theoretically, results for the spin-orbit interactions are presented. Results are used for describing

$$R_0 = A(\mu\, \omega_0(E_0, T))^{\frac{3}{2}} \exp\left(-\mu \frac{3}{4}\right). \qquad (21)$$

The temperature-dependent of exotic electron-hole bound state in Molybdenum disulfide (or moly) QDs are currently fascinating subjects in SQDs physics. Therefore, based on Eqs. (17)-(19) we calculate the mass spectrum of moly QDs with and without spin-orbit interactions at the finite temperature that corresponds to the in-plate directions of the dielectric constant ($\varepsilon_{r||}$). The mass spectrum of exciton in the 1s, 2s, and 3s states for Molybdenum disulfide quantum dots as a function of temperature based on spin-orbit interactions is presented in Fig. 1. We may conclude that our current results as presented in Fig. 1, are in good agreement with currently available experimental data for all states of exotic exciton bound state in nano quantum dots [15]. The present paper proposed a method for theoretically determining the relativistic mass spectrum of exciton at finite a temperature with spin interactions and also energy eigenvalue within the framework of the QFT, QED. An analytic expression was given for masses of exotic systems while considering relativistic corrections.

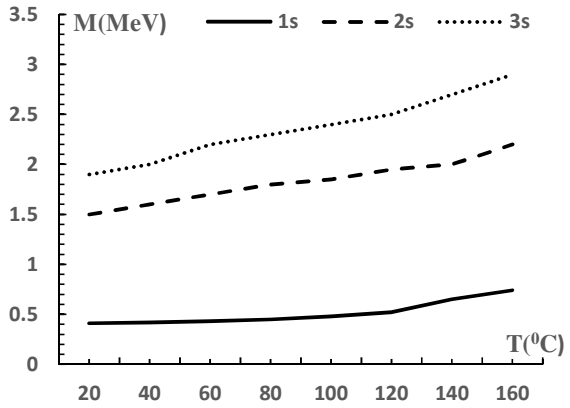

Figure 1. The mass spectrum of exciton in 1s, 2s, and 3s states as a function of temperature based on spin-orbit interactions.

4 Conclusions

The spin-orbit interactions and the relativistic correction to the mass of the electron-hole bound state at a finite temperature in low dimension materials and thin-films under the projective unitary representation are defined theoretically. For determining the spin-orbit interactions at a finite temperature we used the Coulomb potential-temperature relation as an exponential function and modified the radial Schrödinger equation. The formulas show that the high-temperature environment, can be affected by the characteristics of the electron-hole bound state in thin firms.

Based on the description method one can conclude that the theoretical results of this work are expected to define new possibilities and properties of thin films at a finite

temperature which can be used for new materials, microelectronics production, and semiconductor chips.

The obtained theoretical data can be useful in today's researches and can open high perspectives to determine the new characteristics of poly-excitonic systems. We have studied the exciton properties in thin films at finite temperatures. We have calculated the constituent mass of particles based on relativistic correction to the mass. In the above calculations, we have theoretically found that the relativistic behavior on the mass of electron-hole in the exciton system increases with increasing temperature. The mass of the exciton system in 1s, 2s and 3s states as a function of temperature based on spin-orbit interactions are determined and calculated. Based on $\omega_0 = \sqrt{-8\mu\, E(\mu)}$. The ground state amplitude at finite temperature $T > T_0$ will be higher than the amplitude at T_0 (T_0 is the laboratory or room temperature or usually either (T_0 =20÷25 C).

Acknowledgements

This work was supported by scientific programs of the department of physics and engineering sciences, Buein Zahra Technical University.

Funding

This work was supported by scientific programs of the department of physics and engineering sciences, Buein Zahra Technical University under the grant 1999 for developing research.

References

[1] M., Rushka, J., Freericks, "A completely algebraic solution of the simple harmonic oscillator". Am. J. Phys. **88** (11) (2019) 976.

[2] M. Dienykhan, G. Efimov, G. GanboldS, N. Nedelko, "Oscillator Representation in Quantum Physics (Lecture Notes in Physics Monographs)." 1st ed. Springer International Publishing (1995).

[3] D. Fujiwara, "Rigorous Time Slicing Approach to Feynman Path Integrals." 1st ed. Springer International Publishing (2017).

[4] J. Kelley, J. Leventhal, "Ladder Operators for the Harmonic Oscillator. Problems in Classical and Quantum Mechanics." Springer International Publishing (2017).

[5] A. Jahanshir, "Quanto-Optical Effects of Exciton-Polariton System." American Journal of Optics and Photonics, **3** (2015) 89; W. Lucha, F. F. Schoberl, D. Gromes, "Bound states of quarks." Physics Reports, **200** (1991) 127.

[6] R. Gould, "Quantum Electrodynamics. Electromagnetic Processes." 1st ed. Springer-Verlag (2020).

[7] A. Jahanshir, "Relativistic Modification of the Exciton's Mass in Monolayer TMDCs Materials." Journal of Advanced Materials and Processing, **8** (2020) 45.

[8] A. Maireche, "A theoretical investigation of nonrelativistic bound state solution at finite temperature using the sum of modified Cornell plus inverse quadratic potential." Sri Lankan J Physics, **21** (2020)11.

[9] F. Rodríguez-Dueñas, "Optical and transport properties of cuasi-one dimensional electronic systems." Revista de la Academia Colombiana de Ciencias Exactas, Físicas y Naturales, **46** (2018) 150.

[10] R. Meulenberg et al., "Determination of the exciton binding energy in CdSe quantum dots," ACS Nano, **3** (2009) 325.

[11] K. Heyong-chan et al., "New Variational Perturbation Theory Basedonq−Deformed Oscillator." International Journal of Theoretical Physics, **45** (2006) 1017.

[12] A. Taqi, J. Diouri, "A theoretical model for exciton binding energies in rectangular and parabolic spherical finite quantum dots." Semiconductor physics quantum electronics & optoelectronics, **15** (2012) 365.

[13] M. Richard et al., "Exciton-polariton Bose-Einstein condensation: advances and issues", International journal of nanotechnology, **7** (2010) 668.

[14] M. Dharma-wardana et al., "Correlation functions in electron-electron and electron-hole doublequantum wells: temperature, density and barrier-width dependence." arXiv:1901.00895v1 [cond-mat.mes-hall] (2019).

[15] A., Tanveer Karim, et al., " Temperature dependency of excitonic effective mass and charge carrier conduction mechanism in $CH_3NH_3PbI_{3-x}Cl_x$ thin films", Scientific Reports, **11** (2021) 1.