

## Solvation force in hard ellipsoid molecular liquids with rod-sphere and rod- surface interactions

Zahra Firouzi, and Abolghasem Avazapour\*

Department of Physics, University of Yasouj, Yasouj, I.R. Iran

Article history: Received 19 April 2019 Revised 9 August 2019 Accepted 28 August 2019 Available online 29 Aug 2019In a previous study, one of the authors of this work calculated the solvation force of a hard ellipsoid fluid with hard Gaussian overlap potential using hard needle wall interaction and non-linear equation proposed by Grimson- Rickyazen. In the present study, using density functional theory and extended restricted orientation model, the solvation force of hard ellipsoid fluid in the presence of more realistic rod-sphere and rod-surface interactions si calculated. We investigate weak and strong molecule-surface coupling strengths. The colloids distance effects on density profiles are calculated. We could not find the exact or simulation results for comparison. The results in the case $k = 3.0$ are compared with the solvation force of hard Gaussian overlap fluid and hard	ARTICLE INFO	ABSTRACT
Density functional theory needle-wall interaction. The results corresponded, qualitatively.	Article history:Received 19 April 2019Revised 9 August 2019Accepted 28 August 2019Available online 29 Aug 2019Keywords:Solvation forceColloidHard ellipsoid fluidRod - sphere potential,Rod surface - potential,Density functional theory	In a previous study, one of the authors of this work calculated the solvation force of a hard ellipsoid fluid with hard Gaussian overlap potential using hard needle wall interaction and non-linear equation proposed by Grimson- Rickyazen. In the present study, using density functional theory and extended restricted orientation model, the solvation force of hard ellipsoid fluid in the presence of more realistic rod-sphere and rod-surface interactions si calculated. We investigate weak and strong molecule-surface coupling strengths. The colloids distance effects on density profiles are calculated. We could not find the exact or simulation results for comparison. The results in the case $k = 3.0$ are compared with the solvation force of hard Gaussian overlap fluid and hard needle-wall interaction. The results corresponded, qualitatively.

## **1** Introduction

Structure of fluids, are altered near colloidal particles. The force between two colloids is called solvation force. In this paper the solvation force in fluids of hard ellipsoidal molecules with the new particle- wall interactions is calculated. This force has applications in industries, biological fluids, foods, chemical and medical products [1]. Immersed particles with the scale of 10  $\mu$ m to 10 nm create solvation force.

There are different methods to calculate the solvation force. It was calculated for confined polar and nonpolar fluids [2-9] by experimental and Monte Carlo simulation [10] methods. Because of the simplicity it was studied extensively in simple spherical molecular fluids [11-15]. Furthermore it was obtained by Monte Carlo simulations for thin layer of Gay-Berne fluid [16, 17] between two solid walls [18, 19]. In confined hard ellipsoidal molecular fluids with soft particle-wall interactions it has not been much studied theoretically [20].

In one of our previous studies [21], immersed colloidal particles in a fluid of hard ellipsoidal molecules with hard Gaussian overlap (HGO) potential and hard needle-wall (HNW) interaction [22] was studied. In this paper, immersed colloidal particles in a fluid of hard ellipsoidal molecules with more realistic rod-sphere (RSP) and rod-surface potentials (RSUP) [23] are considered. The RSP describes the interaction between a Gaussian ellipsoid and a sphere located in the surface plane. In RSUP each particle effectively interacts with infinity spheres located in the surface plane [23]. The hard ellipsoid (HE) particles do not

interact directly with the substrates, rather another hard ellipsoid is inserted in each particle. As shown in Fig 1, in both models, HE molecules absorb into the walls.



Figure 1. The two particle-wall configurations, (a) RSP and (b) RSUP interactions [23].

We use the density functional theory (DFT) to study the surface influence on liquid crystalline systems. The density functional theories [24, 25] are capable of predicting the phase diagram of liquid crystals [26,27], and also capable of describing the structure near a solid surface and phase transition [25]. Allen [28] showed even the simplest density functional theory namely, the Onsager [29] theory can describe the structure of the surface layer under the influence of external perturbations. This theory has been used to study the thermodynamics of homogeneous and structural properties of inhomogeneous molecular fluids, such as hard ellipsoidal fluids (HE) [30, 31], hard circular cylinders and hard rectangular rods [32,33] and HGO [22] fluids confined between planar walls. Teixeira [31] used the Onsager approximation of density-functional theory with a simple Parsons-Lee re-scaling to study a hybrid-aligned liquid crystal under a very strong confinement. Also, Gurin et al. [34] studied positional ordering of hard cylinders in tubular nanopores and compared the results with classical DFT.

Here we use the hyper-netted chain (HNC) density function proposed by Rickayzen and coworkers [35]. The grand potential of a confined liquid crystal can be written as a functional of the number density of molecules in any direction. After minimizing the grand potential we can obtain the equilibrium number densities, then we obtain the density profile, the average number density [33, 36], and order parameter of a liquid crystal confined between walls before calculating the solvation force.

The extended restricted orientation model (EROM) [37] based on the density functional theory (DFT) and the new direct correlation function (DCF) of Ref. [38] are used to calculate the number density. The RSP and RSUP [23, 39] are used for interaction of colloids and hard ellipsoids with HGO type potential [40] which hare also considered for interaction between ellipsoid molecules. The exact contact distance of hard ellipsoid (HE) molecules is considered instead of the approximate shape parameter of Ref. [39]. For calculation of solvation force the equations which have been proposed by Grimson and Rickyazen, [15, 41] are used. Solvation forces are calculated by using the obtained number densities. Numerical calculations are done by the Fortran software and iteration method [42].

In Section 2, the formalism of solvation force is introduced. In Section 3 the solvation force of a hard ellipsoid fluid with RSP and RSUP interactions is calculated. Finally in Section 4 we obtain the results and present the discussion and conclusions deduced from this work and propose some directions for future studies.

## **2 Formalism of solvation force**

We consider the immersed colloids as rough walls. For confined fluid between these two separated walls by a distance h perpendicular to z direction, following the work of Grimson and Rickayzen, [15] the grand potential difference is;

$$\begin{split} \beta(\Omega[\rho] - \Omega[\rho_0]) &= \beta \int d\vec{r} V_{ext}(z) (\rho(z) - \rho_0) \\ &+ \int d\vec{r} \rho(z) \ln\left(\frac{\rho(z)}{\rho_0}\right) \\ &- \int d\vec{r} \left(\rho(z) - \rho_0\right) \\ &- \frac{1}{2} \int d\vec{r} d\vec{r}' [(\rho(z) - \rho_0)] c(\vec{r}) \\ &- \vec{r}') [\rho(z') - \rho_0], \end{split}$$

where  $\rho(z)$ ,  $\rho_0$  and  $V_{ext}(z)$  are the inhomogeneous one particle density, the homogeneous one particle density, and the external potential respectively. Also  $\beta = (1/k_BT)$  and  $c(\vec{r} - \vec{r}')$  are inverse temperature and direct correlation function. The pressure P(h) that acts upon planar surfaces is defined by [15]

$$\beta P(h) = -\frac{\partial (\Delta \Omega)}{\partial h},\tag{2}$$

where h is the distance of two colloids. Hence, for studying the interaction of two colloids, we need the pressure which arises from the fluid outside the colloids. This pressure will be the same as for two colloids separated by an infinite distance. Thus the total force per unit area which acts to separate the colloids will be

$$f(h) = P(h) - P(\infty) \quad . \tag{3}$$

With this definition, positive f(h) corresponds to a repulsive interaction and negative f(h)corresponds to an attractive interaction. Stable equilibrium separations, therefore, are those at which f(h) passes through zero with a negative spatial gradient.

When minimizing Eq. (1), using the Hyper Netted-Chain (HNC) approximation [35, 43], and differentiating Eq. (1) with respect to the distance h in a confined liquid, the solvation force can be obtained as follows [15]:

$$f(h) = \frac{\rho(h) - \rho(\infty)}{\beta} \quad , \tag{4}$$

where  $\rho(h)$  is one particle density at the position of colloids. By using the Percus-Yevick approximation of density profile, the solvation force is given by:

$$f(h) = \frac{\rho(h)^2 - \rho(\infty)^2}{2\beta\rho_0}$$
(5)

Equations (4) and (5) are called non-linear and linear solvation forces in the literature [15, 41].

# **3 Calculation of solvation force in ellipsoidal molecular fluid**

In real confined ellipsoid fluids, particles can penetrate into the walls, so it can be a soft wall model [19, 22, 23]. For the ellipsoidal molecular liquid as a solvent, the particle-wall interaction has been modeled using RSP and RSUP interactions, where the particlessurface interaction is represented by an ellipsoid, a reduced length,  $k_s$ , located at the center of the particle.  $k_s$  is the determined degree of substrate penetrability [22]. These potentials are convenient interactions for the study of substrate effects on the alignment of liquid crystal molecules. The degree of surface penetrability is increased by decreasing  $k_s$ . The reduced length,  $k_s$  varies between the limits [0:k]. The elongation k is the ratio of the length 2aand the breath 2b of particles, k = 2a/2b. 2b, the length of the minor axis of the molecules, is assumed to be unit length. As Fig 1 and Refs. [22, 23] show, the molecule- wall interaction potential is written as

$$V\left(z_{i},\theta_{\alpha}\right) = \begin{cases} 0 & IF \\ \infty & IF \end{cases} \quad \begin{vmatrix} z_{\alpha}-z_{i} \end{vmatrix} \ge \sigma_{W}\left(k_{s},\theta_{\alpha}\right) \\ |z_{\alpha}-z_{i}| < \sigma_{W}\left(k_{s},\theta_{\alpha}\right) \end{cases}$$
(6)

where  $z_i$ , is position of the wall,  $\sigma_W$  is the normal distance between center of the reduced ellipsoid and

sphere in planar substrate or surface walls. The coordinates  $(z_{\alpha}, \theta_{\alpha})$  refer to the position and orientation of the molecules. For RSP and RSUP interactions, the contact distances are given by [23]:

$$\sigma_{w}^{HGO-Sphere} = \sigma_{0} \left[ \left( 1 - \chi_{s} \cos^{2}\left(\theta_{\alpha}\right) \right)^{-1/2} - \frac{1}{2} \right], \quad (7)$$

and

$$\sigma_{w}^{HGO-Surface} = \sigma_{0} \left[ \left( \frac{1 - \chi_{s} \sin^{2}(\theta_{\alpha})}{1 - \chi_{s}} \right)^{1/2} - \frac{1}{2} \right], \quad (8)$$

where  $\chi_s = (k_s^2 - 1)/(k_s^2 + 1)$  is anisotropy of the reduced ellipsoids. These contact functions have been used as the basis for the particle-substrate interactions. In RSP, the surface, as viewed by any particle, was taken to be represented by a sphere located in the surface plane but with the same *x*-and *y*-coordinates as those of the particles in ref [23]. With RSUP, each HE particle effectively interacts with a planar continuum rather than a single sphere [23].

The interaction between two ellipsoidal particles by means of the HGO potential is given by [40]

$$V\left(\vec{r}_{ij},\hat{\omega}_{i},\hat{\omega}_{j}\right) = \begin{cases} \infty & IF \quad r_{ij} \le \sigma\left(\vec{r}_{ij},\hat{\omega}_{i},\hat{\omega}_{j}\right) \\ 0 & IF \quad r_{ij} > \sigma\left(\vec{r}_{ij},\hat{\omega}_{i},\hat{\omega}_{j}\right) \end{cases}$$
(9)

where  $\hat{\omega}_i = (\theta_i, \varphi_i)$  describes the orientation of the major axis of particle *i* and  $\hat{r}_{12} = \vec{r}_{12}/r_{12}$  is a unit vector along the line connecting the centers of the two particles, and  $\sigma(\vec{r}_{ij}, \hat{\omega}_i, \hat{\omega}_j)$  is given by [35]

$$\sigma(\vec{r}_{12},\omega_1,\omega_2) = 2b \begin{bmatrix} \frac{1-\chi[(\hat{r}_{12}\cdot\hat{\omega}_1)^2 + (\hat{r}_{12}\cdot\hat{\omega}_2)^2]}{1-\chi^2(\hat{\omega}_1\cdot\hat{\omega}_2)} \\ -\frac{2\chi^2(\hat{r}_{12}\cdot\hat{\omega}_1)(\hat{r}_{12}\cdot\hat{\omega}_2)(\hat{\omega}_1\cdot\hat{\omega}_2)}{1-\chi^2(\hat{\omega}_1\cdot\hat{\omega}_2)} \\ +\frac{\lambda[(\hat{r}_{12}\cdot\hat{\omega}_1)^2 + (\hat{r}_{12}\cdot\hat{\omega}_2)^2]^2}{1-\chi^2(\hat{\omega}_1\cdot\hat{\omega}_2)} \end{bmatrix}^{-1/2}, \quad (10)$$

Where  $\chi$  is anisotropy of the molecule versus *k* :

$$\chi = \frac{k^2 + 1}{k^2 - 1},\tag{11}$$

and  $\lambda$  is obtained by

$$\lambda = 1 - \chi - \frac{4b^2}{(a+b)^2} \,. \tag{12}$$

Equation (10) predicts the exact contact distance of HE molecules and Eq. (13) of Ref. [39] is the approximate contact distance of HGO molecules. From references [36, 37] each molecule can be aligned in  $N = 2m^2$  different directions,  $\hat{\omega}_{\alpha\beta} = (\theta_{\alpha}, \varphi_{\beta})$  where

 $\alpha = 0, 1, \dots m - 1$ ,  $\cos \theta_{\alpha} = -1 + \frac{2\alpha + 1}{m}$ , (13)

and

$$\varphi_{\beta} = \beta \frac{\pi}{m}$$
 ,  $\beta = 0, 1, \dots 2m - 1$  . (14)

These allowed directions are uniformly distributed over the surface of a sphere, so equal weighting can be assigned to them. Using the EROM model [37] for molecular liquids, the grand potential, Eq. (1), changes to

$$\frac{\beta \Omega \left[\rho_{\alpha\beta}(z)\right]}{A} = \sum_{n=1}^{\infty} \int_{-h_{\alpha}/2}^{h_{\alpha}/2} dz \rho_{\alpha\beta}(z) \left[ ln \left( \frac{N \rho_{\alpha\beta}(z)}{\rho_{B}} \right) - 1 \right] \\
+ \beta \sum_{\alpha\beta} \int_{-h_{\alpha}/2}^{h_{\alpha}/2} dz \rho_{\alpha\beta}(z) V_{e}(z, \omega_{\alpha\beta}) \\
- \frac{1}{2} \sum_{\alpha\beta} \sum_{\gamma\delta} \int_{-h_{\alpha}/2}^{h_{\alpha}/2} dz_{1} \int_{-h_{\alpha}/2}^{h_{\alpha}/2} dz_{2} C_{\alpha\beta\gamma\delta}(z_{1} - z_{2}) \\
\times \left( \rho_{\alpha\beta}(z_{1}) - \frac{\rho_{B}}{N} \right) \left( \rho_{\alpha\beta}(z_{1}) - \frac{\rho_{B}}{N} \right), \quad (15)$$

where A is the area of the walls and

$$C_{\alpha\beta\gamma\delta}(z_1 - z_2) = \frac{1}{A} \int dx_1 dx_2 dy_1 dy_2 C(\vec{r}_1, \hat{\omega}_1, \vec{r}_2, \hat{\omega}_2, \rho_B), \qquad (16)$$

where  $C(\vec{r}_1, \hat{\omega}_1, \vec{r}_2, \hat{\omega}_2, \rho_B)$  is the DCF of a homogeneous fluid.

Minimizing Eq. (15) with respect to the density with no external potential other than the hard ellipsoidwall, gives a coupled integral equation [36,37]:

$$\rho_{\alpha 0}(z) = \frac{\rho_B}{N} \exp\left[\sum_{\gamma=0}^{m-1} \int_{-h_{\alpha}/2}^{h_{\alpha}/2} dz_1 \left(\rho_{\gamma 0}(z_1) -\frac{\rho_B}{N}\right) \sum_{\delta=0}^{2m-1} c_{\alpha 0\gamma \delta}(z_1-z)\right], \quad (17)$$

Where  $\rho_{\alpha 0}$  is the number density of molecules for any given value of  $\varphi$  and  $\theta_{\alpha}$  where

$$h_{\alpha}/2 = h/2 - \sigma_{W}. \tag{18}$$

For calculating  $C_{\alpha 0\gamma \delta}(z_1 - z)$  of hard ellipsoids, we use the proposed DCF of Refs [38, 44] as follows

$$C(\vec{r}_{12}, \widehat{\omega}_1, \widehat{\omega}_2) = C_{PY-Roth} \left( \frac{|\vec{r}_1 - \vec{r}_2|}{\sigma(\vec{r}_{12}, \widehat{\omega}_1, \widehat{\omega}_2)} \right) [1 + \alpha' P_2(\widehat{\omega}_1 \cdot \widehat{\omega}_2)],$$
(19)

where  $\alpha'$  is a parameter and  $P_2(\hat{\omega}_1, \hat{\omega}_2)$  is the second Legendre polynomial.  $C_{PY-Roth}\left(\frac{|\vec{r}_1 - \vec{r}_2|}{\sigma(\vec{r}_{12}, \hat{\omega}_1, \hat{\omega}_2)}\right)$  was introduced in Ref.

[38]. The total needed density for calculation of solvation force is given:

$$\rho_{tot}(z) = 2m \sum_{\alpha=0}^{m-1} \rho_{\alpha 0}(z)$$
<sup>(20)</sup>

In our calculations, we choose k = 3.0 as it was used in the simulations of refs [22, 23]. For the most commonly used elongation of k = 3.0 HGO fluid, the transition occurs for  $\rho \approx 0.30$  with slight system size dependence [23]. Hence, we chose  $\rho_B = 0.24$  and 0.32 as the bulk isotropic and nematic densities of confined HE liquid, respectively.

We solve integral Eq. (17) for N = 512 by using an iterative method and use Eq. (20) for the total one particle densities. As mentioned in the literature [18, 22, 33, 36, 39], the density profiles,  $\rho_{tot}(z)$ , provide information about the layering in the confined fluid through the location number and height of its peaks. The tendency of confined liquids to orient in a particular direction when in contact with the container walls is called the anchoring phenomenon. The anchoring induced by an interface with interactions, is known as homeotropic, tilt or planar. In homeotropic and planar arrangements, the particles are perpendicular and parallel to the surfaces respectively [18, 22]. Between the two arrangements, lies the possibility of substrateinduced tilt. As mentioned in Refs. [33, 36, 39] and confirmed in the present calculations, the cases,  $k_s = 0.0$  and  $k_s = 3.0$ , result homeotropic, tilt or planar arrangements. For the limited case,  $k_s = 0.0$ , the number density profiles for HE confined fluid with  $h \ge 0.0$  up to 6 times of molecular elongation, are calculated. For the case  $k_s = 0.0$  of RSP and RSUP, in region  $h \approx 0.3$ , there is one monolayer between the two separated walls. For the two cases, in region h < 0.3,  $\rho(h) \rightarrow 0.0$ . Also for case,  $k_s = 3.0$ , h = 1.0 up to six times of molecular elongation, density profiles are calculated. The case RSUP with  $k_s = 3.0$ , corresponds to the hard particle-hard wall interaction. Hence, for h < 1.0 the layer is squeezed out,  $\rho(h) \rightarrow 0.0$  and from Eq. (4),  $\beta f(h) \rightarrow -\rho(\infty)$  is attractive. For  $1.0 \leq h < 1.3$ , there is one monolayer between the two separated walls. For all h values, the case RSP with  $k_s = 3.0$ , corresponds to tilt anchoring. Hence, for h < 1.4 the layer is squeezed out,  $\rho(h) \rightarrow 0.0$  $\beta f(h) \rightarrow -\rho(\infty)$  is attractive. and For  $1.4 \,\tilde{<}\, h < 1.78$ , there is one monolayer between the two separated walls. All mentioned results in layering of molecules are confirmed in solvation force profiles, see Figs 2-5.

From Eq. (4), by using the calculated results of number densities,  $\rho(h)$ , the solvation force with RSP and RSUP interactions are obtained. The solvation forces for HE confined model with RSP and RSUP

interactions are shown in Figs 2 and 3 for limited cases,  $k_s = 0.0$ ,  $k_s = 3.0$  and the constant bulk densities. The effects of two contact functions in isotropic and nematic bulk densities are shown.



Figure 2. Solvation force for confined systems of HE particles as a function of  $h^* = h/2b$  for k = 3.0, (a)  $\rho_B = 0.24$  and (b)  $\rho_B = 0.32$  using the RSP interaction. The solid and doted curves are  $k_s = 3.0$  and  $k_s = 0.0$  (our calculations), respectively. The effect of RSP interaction variation for solvation force of isotropic and nematic HE fluid are shown.



Figure 3. Solvation force for confined systems of HE particles as a function of  $h^*$  for k = 3.0 and  $k_s = 3.0$ , (a)  $\rho_B = 0.24$  and (b)  $\rho_B = 0.32$  using RSP and RSUP interactions. The solid and dashed curves are RSP and RSUP (our calculations), respectively. For RSP and RSUP interactions the effect of bulk density variation for solvation force of planar and tilted arrangements at  $k_s = 3.0$  are shown.

In Figs 4 and 5 the effects of number densities on solvation forces due to different interactions with  $k_s = 0.0$  and  $k_s = 3.0$  are shown.



Figure 4. Solvation force for confined systems of HE particles as a function of  $h^*$  for k = 3.0 and  $k_s = 0.0$ , using RSP and RSUP interactions in different bulk densities. The solvation forces for these two models are shown for  $k_s = 0.0$  and different bulk densities.



Figure 5. Solvation force for confined systems of HE particles as a function of  $h^*$  for k = 3.0,  $k_s = 3.0$ ,  $\rho_B = 0.32$  using RSUP and HNW potentials. The solid, and dashed curves are RSUP (our calculations) and HNW given in [21], respectively.

We could not find the experimental or theoretical data for solvation force of hard ellipsoidal molecular fluids with these two interactions. Hence, in Fig 6 the solvation force profile in  $k_s = k$  as a case of RSUP interaction, is compared with the results of HNW interaction of Ref. [21]. The results are in qualitative agreement.



Figure 6. Solvation force for confined systems of HE particles as a function of  $h^*$  for k = 3.0,  $k_s = 3.0$ ,  $\rho_B = 0.32$  using RSUP and HNW potentials. The solid, and dashed curves are RSUP (our calculations) and HNW given in [21], respectively.

#### 4 Results and discussion

The solvation force of confined HE fluid between colloids has been studied using the density functional theory presented in Section 3. The RSP and RSUP interactions between colloids and ellipsoid molecules are considered. In our calculations we have used the EROM model and nonlinear Grimson– Rickayzen formalism of the solvation force. In all solvation force figures, the forces are oscillatory functions. These decaying oscillatory forces variy between attraction and repulsion forces.

As the bulk density increases, heights and depth of oscillations are increased for all reduced molecule lengths. Also as the distance of colloids increases, oscillations decay away from each colloid. In particular, the figures show oscillatory behavior with periodicity correlating with size of the molecules and magnitude, which decays within a few molecular layers. In the cases

 $k_s = 0.0$  and  $k_s = k$  of RSUP the peak to peak separation of force oscillations corresponds to  $\approx k$  and  $\approx 2b$  respectively. These results agree with homeotropic and planar arrangements [23] of a confined HE fluid with RSUP interaction. The maximas (repulsive parts) correspond to the wellformed layer configurations while the minimas (attractive parts) occur when wall separation is not enough for well-formed layering of HE fluids. Stable separations correspond to the h values where f(h) switches from positive (repulsive) to negative (attractive). In the two interactions, for elongation, k = 3.0, the cases for which  $k_s = 0.0$ correspond to the homeotropic arrangement [23]. Thus solvation force oscillations of RSP and RSUP in the case,  $k_s = 0.0$ , corresponds to homeotropic arrangement. Conversely in the case  $k_s = k$  for RSUP, planar alignment develops, so the number of oscillations increases and their spatial period decreases.

In Figs 2(a) and 2(b) the effect of RSP interaction variation for solvation force of isotropic and nematic HE fluid are shown, respectively. Also in Figs 3(a) and 3(b) the RSUP interaction variation are shown. For RSP and RSUP cases, the first maximum shifts away from the walls as the anchoring changes from homeotropic ( $k_s = 0.0$ ) to tilt or planar ( $k_s = k$ ). The observed peak-peak separation distance of RSP at  $k_s = k$  is not appropriate for either homeotropic or planar anchoring states. The peak-peak separations correspond to tilted arrangement of confined HE molecules. In planar and homeotropic arrangements, the main peaks are located at distances  $\approx 2b$  and  $\approx 0.2 \sim 0.3$  from the walls and correspond to the first molecules layering. The amplitude of maxima to adjacent minima in bulk nematic is longer than for the isotropic case. For the planar case, oscillations in f(h) of period ~1.1 in the nematic phase indicate stratification which corresponds to integer numbers of planar layers in the h interval. For the homogeneous case of RSUP, the oscillations are of longer period and weaker amplitude, indicating weaker orientational coupling in this geometry.

In Figs 4(a) and 4(b) the effect of bulk density variation for solvation force of planar and tilted arrangements at  $k_s = 3.0$  are shown. The increase in bulk density from isotropic to nematic, increases the height of maxima and depth of minima in planar and tilted arrangements. The heights and depths of oscillations of RSUP in isotropic and nematic densities are higher than for the RSP case. Also peak to peak separations of force oscillations increase from RSUP to RSP. This increment corresponds to tilted alinement of RSP.

The amplitude of oscillations tends to zero in  $h \ge 4.0$  and  $h \ge 7.0$  for planar isotropic and nematic cases of RSUP, respectively. For tilted isotropic and nematic cases of RSP, zero amplitude occurs in  $h \ge 8.0$  and  $h \ge 10.0$ , respectively.

The solvation forces for these two models are shown in Fig 5 for  $k_s = 0.0$  and different bulk densities. For homeotropically isotropic and nematic cases of the two interactions, the amplitudes tend to zero in h > 3k and h > 4k, respectively. These behaviors correspond to layering and orientational ordering of confined HE molecules. Peaks of isotropic cases shifted away from the wall and broadened.

To some extent, it is reasonable to compare our results with the results of Ref. [21], because simulation or experimental results of solvation force for hard ellipsoid fluids with RSP or RSUP interaction were not available. For the case k = 3.0,  $k_s = 3.0$ , as a RSUP interaction, our results are compared with results of confined HGO fluids with HNW interaction in Fig 6. The main reasons for the discrepancy in Fig 6 may be due to molecule-surface interaction strength difference and molecule- molecule interaction difference.

## **5** Conclusions

The conclusions are summarized as follows:

(a) In real confined ellipsoid fluids, particles can absorb into the walls.  $k_s$  is determined as the degree of substrate penetrability [22, 23]. The RSP and RSUP models are alternative interactions for studying these confined fluids.

(b) In homeotropically anchored cases ( $k_s = 0.0$ ), the formation of layering requires more space than in planar and tilt anchored cases. The distance  $\Delta h$  between the first and second maxima or minima in solvation force corresponds roughly to the large `diameter' of a HE molecule.

(c) In RSP tilt anchored cases  $(k_s = k)$ , for isotropic and nematic bulk densities, the formation of layering requires more space than in RSUP planar anchored cases. The distance  $\Delta h$  between two consecutive maxima or minima in solvation force of RSP is  $1.4 < \Delta h < k$ .

(d) In RSUP nematic planar cases, the periods of force oscillations close to the walls are approximately smaller than those farther from the walls. This effect depends on influence of the layers near the walls on adjacent molecules in these cases.

(e) The results show that factors such as particlewall interactions and bulk number density of HE fluids can have significant effect on solvation forces of the confined fluids.

(f) The present calculations showed that the extended restricted orientational model of HNC density functional theory, by using Eq. (10) instead of Eq. (13) of Ref. [39], can be usefully applied to confined liquids comprised of HE particles with RSP and RSUP particle-wall potentials. Finally, EROM could qualitatively predict the solvation forces corresponding to the behavior of homeotropic, tilt, and planar arrangements of the HE confined liquid.

We can calculate the solvation force of confined Gay-Berne fluids with realistic particle-substrate interactions; using this theoretical model and simulation methods. Also we can calculate the solvation force of confined HE using simulation methods. We will do these in future works.

### Acknowledgments

The authors would like to thank the Yasouj University Research Council for financial support for this work [grant number Gryu-89131110].

### References

[1] J.N. Israelachvili. Intermolecular and Surface Forces. Second Ed. Academic Press, London, 1992.

[2] R.G. Horn, J.N. Israelachvili, E. Perez, "Forces Due to Structure in a Thin Liquid Crystal Film." Journal de Physique, **42** (1981) 39.

[3] M.L. Gee, P.M. McGuiggan, J.N. Israelachvili, et al., "Liquid to Solid-Like Transitions of Molecularly Thin Films under Shear." Journal of Chemical Physics, 93 (1990) 1895.

[4] J. Klein, E. Kumacheva, "Simple liquids confined to molecularly thin layers. I. Confinement-induced liquid-to-solid phase transitions." Journal of Chemical Physics, **108** (1998) 6996.

[5] L.J. Demirel, S. Granick," Origins of solidification when a simple molecular fluid is confined between two plates." Journal of Chemical Physics, **115** (2001) 1498.

[6] R. Lim, S.F.Y. Li, S.J. O'Shea, "Solvation forces using sample-modulation atomic force microscopy." Langmuir, **18** (2002) 6116.

[7] N.N. Gosvami, "Solvation Forces and Contact Mechanics at the Nanometer Scale in Molecular Liquids." Ph. D Thesis, Singapore: National University of Singapore; 2008.

[8] N.N. Gosvami, W. Hofbauer, S.J. O'Shea, S.K. Sinha, "Solvation and squeeze out of hexadecane on graphite." Journal of Chemical Physics, **126** (2007) 214708.

[9] S. de Beer, D. van den Ende, F. Mugele, "Dissipation and oscillatory solvation forces in confined liquids studied by small-amplitude atomic force spectroscopy." Nanotechnology, **21** (2010) 325703.

[10] A. Malani, K.G. Ayappa, "Confined fluids in a Janus pore: influence of surface asymmetry on structure and solvation forces." Molecular Simulation, 38 (2012) 1114.

[11] P. Attard, J.L. Parker, "Oscillatory Solvation Forces: a Comparison of Theory and Experiment." Journal of Chemical Physics, **96** (1992) 5086.

[12] C.N. Patra, S.K. Ghosh, "Weighted-densityfunctional theory of solvation forces in liquids." Physical Review E, **49** (1994) 2826.

[13] M. Moradi, M. Kavosh Tehrani M, "Solvation forces in Lenard Jones and Stockmayer fluid at various temperatures." Iranian Journal of Science and Technology, Transactions A: Science, **25** (2001) 377.

[14] K. Yang, Y. Lin, X. Lu, A.V. Neimark, "Solvation forces between molecularly rough surfaces." Journal of Colloid and Interface Science, **362** (2011) 382.

[15] M.J. Grimson, G. Rickayzen, "Linear and nonlinear theories of solvation forces in fluids." Molecular Physics, **42** (1981) 767.

[16] J.G. Gay, B.J. Berne, "Modification of the overlap potential to mimic a linear site– site potential." Journal of Chemical Physics, **74** (1981) 3316.

[17] R. Everaers, M.R. Ejtehadi, "Interaction potentials for soft and hard ellipsoids." Physical Review E, **67** (2003) 041710.

[18] M. Schoen, T. Gruhn, D.J. Diestler, "Solvation forces in thin films confined between macroscopically curved substrates." Journal of Chemical Physics, **109** (1998) 301.

[19] T. Gruhn, M. Schoen, "A grand canonical ensemble Monte Carlo study of confined planar and homeotropically anchored Gay-Berne films", Molecular Physics, **93** (1998) 681. [20] S. Kondrat, A. Poniewierski, L. Harnau, "Orientational phase transition and the solvation force in a nematic liquid crystal confined between inhomogeneous substrates." European Physical Journal E, **10** (2003) 163.

[21] Z. Ordibeheshti, A. Aavazpour, E. Sadeghi, et al., "Solvation force in hard ellipsoid fluids with HNW interaction." Molecular Physics, **114** (2014) 2023.

[22] F. Barmes, D.J. Cleaver, "Computer simulation of a liquid-crystal anchoring transition." Physical Review E, **69** (2004) 061705.

[23] F. Barmes F, D.J. Cleaver, "Using particle shape to induce tilted and bistable liquid crystal anchoring." Physical Review E, **71** (2005) 021705.

[24] J.P. Hansen, I.R. Mc Donald. Theory of Simple Liquids. 4th Ed. Academic Press, London, 2013.

[25] M. Moradi, B. B. Ghotbabadi and R. Aliabadi, "The study of uniaxial-biaxial phase transition of confined hard ellipsoids using density functional theory." International Journal of Modern Physics C, **28** (2017) 1750068.

[26] E. Velasco and L. Mederos, "A theory for the liquid-crystalline phase behavior of the Gay–Berne model." Journal of Chemical Physics, **109** (1998) 2361.

[27] E. Velasco, A. M. Somoza and L. Mederos, "Liquid-crystal phase diagram of the Gay–Berne fluid by perturbation theory." Journal of Chemical Physics, **102** (1995) 8107.

[28] M. P. Allen, "Molecular simulation and theory of liquid crystal surface anchoring." Molecular Physics, **96** (1999) 1391.

[29] L. Onsager, "The effects of shape on the interaction of colloidal particles." Annals of the New York Academy of Sciences, **51** (1949) 627.

[30] P. Kalpaxis and G. Rickayzen, "Structure of a confined fluid of hard ellipsoids." Molecular Physics, **80** (1993) 391.

[31] P.I.C.Teixeira, "Nematic liquid crystal order reconstruction in ultraconfinement, from density-functional theory." Liquid Crystals, **43** (2016) 1526.

[32] G. Rickayzen, "Phase transitions in molecular fluids: The restricted orientation model." Molecular Physics, **80** (1993) 1093.

[33] R. Aliabadi1, P. Gurin, E. Velasco and S. Varga, "Ordering transitions of weakly anisotropic hard rods in narrow slit-like pores." Physical Review E, **97** (2018) 012703.

[34] P. Gurin, S. Varga, Y. Martínez-Ratón and E. Velasco, "Positional ordering of hard adsorbate particles in tubular nanopores." Physical Review E, **97** (2018) 052606.

[35] G. Rickayzen, "A model for the study of the structure of hard molecular fluids." Molecular Physics, **95** (1998) 393.

[36] M. Moradi, Richard J. Weatley, A. Avazpour, "Density functional theory of liquid crystals and surface anchoring." Physical Review E, **72** (2005) 061706.

[37] M. Moradi, Richard J. Weatley, A. Avazpour, "Density profile and order parameter of a hard ellipsoidal fluid confined to a slit." Journal of Physics: Condensed Matter, **17** (2005) 5625.

[38] A. Avazpour, M. Moradi, "The direct correlation functions of hard Gaussian overlap and hard ellipsoidal fluids." Physica B: Condensed Matter, **392** (2007) 242.

[39] A. Avazpour, L. Avazpour, " Density functional theory of liquid crystals and surface anchoring: Hard Gaussian overlap-sphere and hard Gaussian overlap-surface potentials." Journal of Chemical Physics, **133** (2010) 244701.

[40] M. Moradi, A. Avazpou, "Density profiles of a hard Gaussian overlap fluid between hard walls." International Journal of Modern Physics B, **19** (2005) 1717.

[41] M.J. Grimson, G. Rickayzen, "Solvation forces in charged fluids." Molecular Physics, **45** (1982) 221.

[42] W.H. Press, S.A. Teukolsky, W.T. Vetterling, et al. Numerical Recipes in Fortran 90. Second Ed. Cambridge University Press, New York (NY), 1992.

[43] L.S. Ornestin, F. Zernike, "Accidental deviations of density and opalescence at the critical point of a single substance." KNAW, Proceedings, **17** (1914) 793.

[44] J.F. Marko, "First-order phase transitions in the hard-ellipsoid fluid from variationally optimized direct pair correlations." Physical Review A, **39** (1989) 2050.